

National University of Singapore
Department of Mathematics

Semester II, 2000/2001

GEM1417K Mathematical Thinking

MID-TERM TEST

Name:

(PLEASE PRINT)

Tutorial Date & Time:

Answer **ALL THREE (3)** questions.

1. (5 marks) Explain the term **speculation**, and the roles it plays in mathematical thinking. Give **an** example to support your discussion.

Answer:

Answer:

2. (5 marks) Starting with the letter b (step 1), consider the rewriting rules

$$b \rightarrow a; \quad a \rightarrow ab.$$

For example, the first few ‘words’ formed by the above rewriting rules are

$$b, a, ab, aba, abaab, abaababa, \dots$$

- (a) In *anyone* of the words so obtained, can the letter b be adjacent to another b ? That is, can we find a word of the form $\dots bb \dots$? Justify your answers.
- (b) Let $F(n)$ be the number of letters in the n -th word (n is a positive integer). For instance,

$$F(1) = 1, F(2) = 1, F(3) = 2, F(4) = 3, F(5) = 5, F(6) = 8, \dots$$

Prove that $F(n + 1) = F(n) + F(n - 1)$ for *all* positive integers $n \geq 2$.

Answer:

Answer:

3. (a) (5 marks) Find a formula (in terms of n) for the sum

$$1 + 3 + 5 + \cdots + (2n + 1).$$

[*Hint:* Apply the idea of ‘retrograding’ $(2n + 1) + \cdots + 5 + 3 + 1$.] Here n is a positive integer. You are NOT required to prove the answers. Instead, *provide a geometric interpretation of the result.*

- (b) (5 marks) Assume that you do NOT know the formula for the sum $1^2 + 2^2 + \cdots + n^2$ (that is, you are *not* allowed to use the formula). Formulate a conjecture on the form of a formula (in terms of the positive integer n) for the sum

$$1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2.$$

Prove your assertions. (Beware of the first term $1^2 = (2 \cdot 0 + 1)^2$. That is, it corresponds to $n = 0$.)

Answer:

Answer:

Please write your name on any additional sheets and staple them to the test paper.