RUNNING A BUSINESS

CURVE SKETCHING AS A WAY TO GET RICH.

Suppose that you run a business selling chicken rice. The capital of your business, the total amount invested in it, will be a function of time, c(t). Let's suppose that the business is doing well and making a profit. What do we mean by "doing well"? What we mean is this: the ratio of the profit per year, p, to c(t), is large. (Even though $ 1 million per year seems a lot to most of us, it would be regarded as a very bad profit for Microsoft Corp, which is worth many billions. On the other hand, $ 1 million per year would be highly acceptable for a chicken rice stall! The point is that p (measured in dollars per year) is not a measure of success: the measure of success is the ratio p/c(t).) Let's suppose that our chicken rice business is making a profit and that its "success" is a constant in time, equal to S.

Suppose first that we take all of the profit and put it back into the business. Then all of the profit per year, p, goes back into increasing c(t), so p is given by dc/dt. (Remember: the derivative is the rate of increase.) So if the success is S, we have

$$S = p/c(t) = \frac{dc}{dt}/c(t), \tag{1}$$

which, by using the chain rule on the composite function ln(c(t)), can be written as

$$S = \frac{d\, ln(c(t))}{dt}. \tag{2}$$

However, we would not really put all of the profit back into the business — there would be no point in a business like that! Suppose instead that we put a fraction of p back into the business — let's say a fraction k. Then only kp dollars per year go towards increasing the capital c(t), so dc/dt is just kp, or p = (dc/dt)/k, and we have

$$S = p/c(t) = \frac{dc}{dt}/kc(t), \tag{3}$$

or

$$Sk = \frac{d\, ln(c(t))}{dt}. \tag{4}$$

Taking the antiderivative, we have

$$ln(c(t)) = Skt + K, \tag{5}$$

where K is a constant. Taking the exponential, we find

$$c(t) = exp(Skt + K) = exp(Skt) \times exp(K). \tag{6}$$

Suppose that the initial capital, when you founded your business, was C. Then c(0) = C, so C = exp(K), so

$$c(t) = Cexp(Skt). \tag{7}$$

How much have you paid out in dividends? Of course, the answer is: all the profit that you DIDN'T put back into the business. Let M(t) be the TOTAL amount of money that your business has paid out up to time t from t = 0. (So of course M(0) = 0.) Since you put a fraction k of your profit p back into the business, the fraction of p you paid out must have been 1-k. So you have been paying out (1-k)p dollars per year, and so

$$\frac{dM}{dt} = (1-k)p. \tag{8}$$

But by definition S = p/c(t), so by equation 7

$$p = Sc(t) = SCexp(Skt), \tag{9}$$

and thus

$$\frac{dM}{dt} = (1-k)SCexp(Skt). \tag{10}$$

Taking the antiderivative gives us

$$M(t) = A + C(k^{-1} - 1)exp(Skt), \tag{11}$$

provided that k is not exactly zero, where A is a constant. (Let's ignore the case k = 0 — it is very simple and is left to you as an exercise.) Since M(0) = 0, we see that $A = -C(k^{-1} - 1)$, and so

$$M(t) = C(k^{-1} - 1)[exp(Skt) - 1], \tag{12}$$

Suppose now that our plan is as follows. We do not propose to spend our entire lives in the chicken rice business; we will spend T years selling chicken rice and then pull out in order to study Engineering. Our total profit, \$, will therefore be

$$\$(k) = M(T) = C(k^{-1} - 1)[exp(SkT) - 1]. \tag{13}$$

Here we think of C, S, and T as fixed numbers, so that $(k) is indeed a function of k. A BUSINESS STRATEGY is simply a choice of k: what fraction of your profit should you put back into the business? Of course, we answer this by MAXIMISING $(k) as a function of k. This is actually a difficult problem, because $(k) is a complicated function, but we can easily solve it by getting a computer to GRAPH $(k). For example, suppose that C = 1 million dollars, S = 1 [remember that S measures the "success" of your business] and T = 5 years, so that

$$\$(k) = (k^{-1} - 1)[exp(5k) - 1]. \tag{14}$$

The graph of this function is as shown [of course we only care about k between 0 and 1].

The vertical axis is measured in units of millions of dollars. Notice that if we pick k to be close to zero [ie we put nothing back into the business] then our total dividends over 5 years are 5 million dollars, which makes sense since S = 1 means that you are making a million dollars a year and you are paying all of it out as dividends. But it is obvious from the diagram that you would be MUCH better off putting about 0.73 of your profits each year back into the business — if you do that you will make almost 14 MILLION

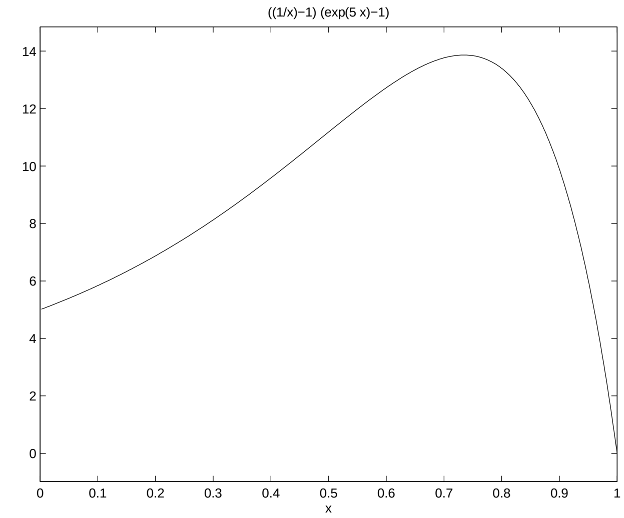

Figure 1: Graph of $\$ = (k^{-1} - 1)[exp(5k) - 1]$

DOLLARS IN DIVIDENDS! Any fraction less or more than this is wasteful. Obviously the figure 0.73 is not something that you would have guessed!

Suppose you are more impatient and you only want to stay in the business for 1 year, T = 1, everything else unchanged. Then

$$\$(k) = (k^{-1} - 1)[exp(k) - 1], \tag{15}$$

and the graph is as follows:

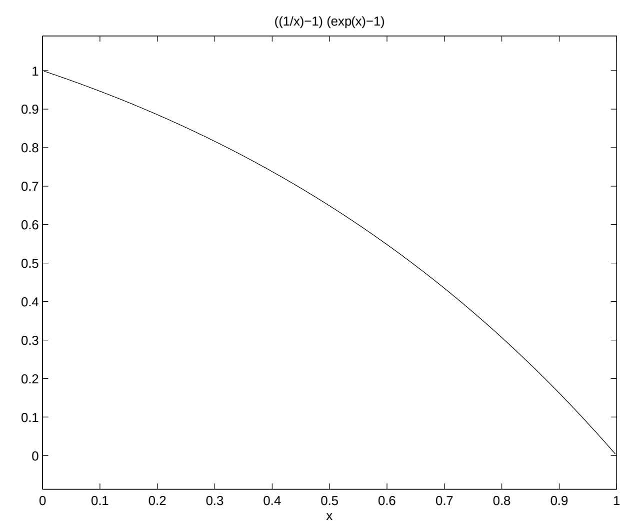

Figure 2: Graph of $\$ = (k^{-1} - 1)[exp(k) - 1]$

Now we see something very different: the maximum of the function occurs at the left end, and the best choice is k = 0. So if you are only going to be in business for one year, you should put NOTHING back into the business; just keep your profit (one million dollars). Interesting question: what is the value of T such that it begins to become worthwhile to start putting some of your profits back into the business? You can answer this by playing with T and plotting. The answer is 2 years!