

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1508 Linear Algebra with Applications

2006-2007 (Semester 2)

Tutorial 1

1. Consider the following linear system

$$\begin{cases} 3x_2 - 6x_3 + 6x_4 + 4x_5 = -5 \\ 3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9 \\ 3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15 \end{cases}$$

- (a) Write down the augmented matrix \mathbf{A} of the linear system.
(b) Use Algorithm 2.9 to find a row-echelon form of \mathbf{A} .
(c) Hence write down a general solution of the linear system.
(d) Use Algorithm 2.10 to find the reduced row-echelon form of \mathbf{A} .
2. Solve for x , y and z in the following linear system

$$\begin{cases} xy - 2\sqrt{y} + 3zy = 8 \\ 2xy - 3\sqrt{y} + 2zy = 7 \\ -xy + \sqrt{y} + 2xy = 4 \end{cases}$$

3. Find the inverse of the following matrix

$$\mathbf{A} = \begin{pmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{pmatrix}.$$

4. For the square matrix \mathbf{A} of order n , the *trace* of \mathbf{A} , denoted by $tr(\mathbf{A})$ is defined to be

$$tr(\mathbf{A}) = a_{11} + a_{22} + \dots + a_{nn}.$$

Show that

- (a) $tr(\mathbf{A}\mathbf{A}^T) = tr(\mathbf{A}^T\mathbf{A})$.
(b) If \mathbf{B} is also a square matrix of order n , show that $tr(\mathbf{B}\mathbf{A}) = tr(\mathbf{A}\mathbf{B})$.
(c) Can you find two matrices \mathbf{A} and \mathbf{B} such that $\mathbf{B}\mathbf{A} - \mathbf{A}\mathbf{B} = \mathbf{I}$?
5. (a) Does there exist an inconsistent linear system with more unknowns than equations?
(b) Does there exist a linear system which has exactly one solution but has more equations than unknowns?
(c) Does there exist a linear system which has exactly one solution but has more unknowns than equations?
(d) Does there exist a linear system which has infinitely many solutions but has more equations than unknowns?

Justify your answers.