

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1508 Linear Algebra with Applications

2006-2007 (Semester 2)

Tutorial 4

1. Let $\mathbf{v}_1 = (2, 1, 0, 3)$, $\mathbf{v}_2 = (1, 2, 5, -1)$ and $\mathbf{v}_3 = (7, -1, 5, 8)$.
 - (a) Show that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set.
 - (b) Is $\text{span}(S) = \mathbb{R}^4$? Does S form a basis for \mathbb{R}^4 ?
 - (c) If your answer to part (b) is no, can you add vector(s) into S (ie. make the set bigger) so that we have a basis for \mathbb{R}^4 ?
2. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be vectors and

$$\begin{aligned} S_1 &= \{\mathbf{a}, \mathbf{b}\} \\ S_2 &= \{\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{c} - \mathbf{a}\} \\ S_3 &= \{\mathbf{a} - \mathbf{b}, \mathbf{b} - \mathbf{c}, \mathbf{c} + \mathbf{a}\} \\ S_4 &= \{\mathbf{a}, \mathbf{a} + \mathbf{b}, \mathbf{a} + \mathbf{b} + \mathbf{c}\} \\ S_5 &= \{\mathbf{a} + \mathbf{b}, \mathbf{b} + \mathbf{c}, \mathbf{a} + \mathbf{c}, \mathbf{a} + \mathbf{b} + \mathbf{c}\} \end{aligned}$$

- (a) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors in \mathbb{R}^3 and suppose $\text{span}(\mathbf{a}, \mathbf{b}, \mathbf{c}) = \mathbb{R}^3$. Which of the sets above also span \mathbb{R}^3 ?
 - (b) If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly independent vectors in \mathbb{R}^n , which of the sets above are also linearly independent?
3. Let $\mathbf{v}_1, \mathbf{v}_2$ and \mathbf{v}_3 be vectors in \mathbb{R}^3 and \mathbf{A} be a 3×3 matrix. For this question, treat all vectors in \mathbb{R}^3 as column vectors.
 - (a) Note that $\mathbf{A}\mathbf{v}_1, \mathbf{A}\mathbf{v}_2, \mathbf{A}\mathbf{v}_3$ are also vectors in \mathbb{R}^3 . Show that if $\mathbf{A}\mathbf{v}_1, \mathbf{A}\mathbf{v}_2, \mathbf{A}\mathbf{v}_3$ are linearly independent vectors, then $\mathbf{v}_1, \mathbf{v}_2$ and \mathbf{v}_3 are also linearly independent.
 - (b) Suppose $\mathbf{v}_1 = (2, 1, -7)^T$, $\mathbf{v}_2 = (-2, -3, -5)^T$, $\mathbf{v}_3 = (1, 2, 4)^T$.
 - (i) Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent.
 - (ii) Pick your favorite 3×3 matrix \mathbf{A} (other than \mathbf{I}_3 and $\mathbf{0}_3$). Determine, for your choice of \mathbf{A} , if $\mathbf{A}\mathbf{v}_1, \mathbf{A}\mathbf{v}_2, \mathbf{A}\mathbf{v}_3$ are also linearly independent.
 - (iii) Experiment with few other choices of \mathbf{A} . See if you can develop a theory on whether the set $\{\mathbf{A}\mathbf{v}_1, \mathbf{A}\mathbf{v}_2, \mathbf{A}\mathbf{v}_3\}$ will (always? never? sometimes?) be linearly independent. Prove your theory, if possible!

4. Determine which of the following statements are true. Justify your answers.
 - (a) Suppose V is a subspace of \mathbb{R}^2 . If $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for V , then it is also a basis for \mathbb{R}^2 .
 - (b) Suppose V is a subspace of \mathbb{R}^3 . If $\{\mathbf{v}_1, \mathbf{v}_2\}$ spans V , then $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for V .

- (c) If $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$ is a basis for a subspace V of \mathbb{R}^n , then $\{c\mathbf{v}_1, c\mathbf{v}_2, \dots, c\mathbf{v}_k\}$ is also a basis for V for all $c \neq 0$.
- (d) There exist subspaces V and W of \mathbb{R}^3 such that $\dim(V - W) = \dim(V) = \dim(W)$. (Refer to Tutorial 3 Question 5 for definition of $V - W$.)
5. For each of the following, find the coordinate vector of \mathbf{v} relative to the basis $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
- (a) $\mathbf{v} = (2, -1, 3)$, $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (2, 2, 0)$, $\mathbf{u}_3 = (3, 3, 3)$.
- (b) $\mathbf{v} = (5, -12, 3)$, $\mathbf{u}_1 = (1, 2, 3)$, $\mathbf{u}_2 = (-4, 5, 6)$, $\mathbf{u}_3 = (7, -8, 9)$.