NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics
MA1508 Linear Algebra with Applications
2006-2007 (Semester 2) **Tutorial 5**

1. (a) For what values of a will the vectors $\boldsymbol{u_1} = (a, 1, 1)$, $\boldsymbol{u_2} = (1, a, 1)$, $\boldsymbol{u_3} = (1, 1, a)$ form a basis for $\mathbb{R}^3$?

   (b) We know that the solution set of any homogeneous linear system $\boldsymbol{Ax} = \boldsymbol{0}$ with n unknowns is always a subspace of $\mathbb{R}^n$. For what values of a is the solution space of

$$\begin{cases} ax_1 + x_2 + x_3 = 0 \\ x_1 + ax_2 + x_3 = 0 \\ x_1 + x_2 + ax_3 = 0 \end{cases}$$

   a line through the origin? a plane containing the origin? the zero space? the entire $\mathbb{R}^3$?

2. Suppose $\{\boldsymbol{a_1}, \boldsymbol{a_2}, \boldsymbol{a_3}\}$ is a basis for a vector space V. Determine if $\{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}\}$ is also a basis for V if

   (a) $\boldsymbol{u_1} = \boldsymbol{a_1}$, $\boldsymbol{u_2} = \boldsymbol{a_1} - \boldsymbol{a_2}$, $\boldsymbol{u_3} = \boldsymbol{a_1} - \boldsymbol{a_2} - \boldsymbol{a_3}$.

   (b) $\boldsymbol{u_1} = \boldsymbol{a_1} + 2\boldsymbol{a_2}$, $\boldsymbol{u_2} = 2\boldsymbol{a_2} + 3\boldsymbol{a_3}$, $\boldsymbol{u_3} = 3\boldsymbol{a_3} - \boldsymbol{a_1}$.

3. Suppose $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are three linearly independent vectors in $\mathbb{R}^n$.

   (a) If $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers such that

$$a_1\boldsymbol{u} + b_1\boldsymbol{v} + c_1\boldsymbol{w} = a_2\boldsymbol{u} + b_2\boldsymbol{v} + c_2\boldsymbol{w},$$

   prove that $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$.

   (b) Hence or otherwise, prove that

$$V = \{\boldsymbol{u} + s\boldsymbol{v} + t\boldsymbol{w} | s, t \in \mathbb{R}\}$$

   can never be a subspace of $\mathbb{R}^n$.

4. Let $S_1 = \{\boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}\}$ (standard basis of $\mathbb{R}^3$) and $S_2 = \{(3, -1, 4), (2, 0, -5), (8, -2, 7)\}$.

   (a) Show that S_2 is a basis of $\mathbb{R}^3$.

   (b) Find the transition matrix from S_1 to S_2.

   (c) For the vector $\boldsymbol{w} = (1, 2, 3)$, find $(\boldsymbol{w})_{S_2}$.

5. Solve the following LP using simplex method.

$$\max 2x_1 - x_2 + 8x_3$$

$$\begin{aligned} \text{subject to} \qquad 2x_3 &\leq 1 \\ 2x_1 - 4x_2 + 6x_3 &\leq 3 \\ -x_1 + 3x_2 + 4x_3 &\leq 2 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$