NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

MA1508 Linear Algebra with Applications

2006-2007 (Semester 2) **Tutorial 6**

1. (a) Find a basis for the (i) row space; (ii) column space; (iii) nullspace of the following matrix.
$$\boldsymbol{A} = \begin{pmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{pmatrix}.$$
   (b) Verify the Dimension Theorem for matrices.
   (c) Extend the basis found in (a) for the row space of $\boldsymbol{A}$ to a basis for $\mathbb{R}^5$. (Extend means add vectors into the set of basis vectors).

2. Let $H = \text{span}\{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ and $K = \text{span}\{\boldsymbol{v_1}, \boldsymbol{v_2}\}$ where
$$\boldsymbol{u_1} = \begin{pmatrix} 5 \\ 3 \\ 8 \end{pmatrix}, \boldsymbol{u_2} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}, \boldsymbol{v_1} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}, \boldsymbol{v_2} = \begin{pmatrix} 0 \\ -12 \\ -28 \end{pmatrix}.$$
   Note that H and K are both planes in $\mathbb{R}^3$ containing the origin and they intersect in a line L that passes through the origin. Find a basis for L.

3. Determine which of the following statements are true and which are false. Justify your answers.
   (a) If $\boldsymbol{Ax} = \boldsymbol{0}$ is a homogeneous linear system and $\boldsymbol{A}$ is a 10×12 matrix, then it is possible that the nullspace of $\boldsymbol{A}$ can be represented by
$$n(\boldsymbol{A}) = \{k\boldsymbol{v} | \boldsymbol{v} \text{ is a non-zero vector}, k \in \mathbb{R}\}.$$
   (b) There is a 5×5 matrix $\boldsymbol{A}$ such that dim(row space of $\boldsymbol{A}$)=dim(column space of $\boldsymbol{A}$)=3 and row space of $\boldsymbol{A}$=column space of $\boldsymbol{A}$.
   (c) There is a 5×5 matrix $\boldsymbol{A}$ such that dim(row space of $\boldsymbol{A}$)=dim(column space of $\boldsymbol{A}$)=3 and row space of $\boldsymbol{A} \neq$ column space of $\boldsymbol{A}$.

4. Suppose $\boldsymbol{A}$ is a $m \times n$ matrix and $\boldsymbol{B}$ is a $n \times p$ matrix.
   (a) Show that $\text{rank}(\boldsymbol{AB}) \leq \text{rank}(\boldsymbol{A})$.
   (b) Hence, or otherwise, show that if $\boldsymbol{P}$ is a $m \times m$ invertible matrix, then $\text{rank}(\boldsymbol{PA}) = \text{rank}(\boldsymbol{A})$.

5. Let G be a digraph of order n. Suppose the adjacency matrix of G is defined as follows:
$$a_{ij} = \begin{cases} 1 & \text{if } |i-j| \text{ is odd;} \\ 0 & \text{otherwise.} \end{cases}$$
   Show that G has no cliques.