ASSIGNMENT 1 (Due 10 September 2012)

1. Write a MATLAB function \([Q, R] = clgs(A)\) that computes the \(QR\) factorization \(A = QR\) of an \(m \times n\) matrix \(A\) with \(m \geq n\) using Gram-Schmidt orthogonalization. The output variables are a matrix \(Q\) with orthogonal columns and a triangular matrix \(R\).

2. Let \(A\) be the matrix

\[
A = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
4 & 5 & 6 & 7 & 8 \\
7 & 8 & 7 & 9 & 10 \\
4 & 2 & 3 & 4 & 5 \\
4 & 2 & 2 & 3 & 4
\end{pmatrix}
\]

Compute the two \(QR\) factorization of \(A\) in MATLAB: by the Gram-Schmidt routine \(clgs\) of question 1, and by MATLAB’s built-in command \([Q, R] = qr(A)\). Compare the results and comment on any differences you see.

3. The \(n \times n\) matrix \(A\) is defined as follows:

\[a_{i,i} = 2, \text{ for } i = 1, \cdots, n; \quad a_{i,i+1} = a_{i+1,i} = -1, \text{ for } i = 1, \cdots, n-1\]

and all other entries are zero.

3.1 The condition number for this matrix takes the form \(c = pn^q\), where \(n\) is the size of the matrix, \(c\) is the condition number, \(p\) and \(q\) are constants. Compute the condition number for the matrix \(A\) for \(n\) from 100 to 1000 with increment 100 using the MATLAB function \(cond\), fit the function \(pn^q\) (graphically) to the set of results you produce and estimate \(p\) and \(q\). Hint: Take logs of both side of the equation and plot log(\(c\)) against log(\(n\)).

3.2 For \(n = 1000\), generate a random vector \(x_0\) of length \(n\) using the MATLAB function \(randn(n,1)\). Define \(b = A \times x_0\), then use the QR factorization (MATLAB function \(qr\)) to solve the linear system \(Ax = b\). Compute the backward error \(\|Ax - b\|/\|b\|\) and the forward error \(\|x - x_0\|/\|x_0\|\). Discuss your finding. Are they consistent with the error analysis that we discussed in class?

3.3 Repeat question 3.2 using the Gram-Schmidt routine \(clgs\) of question 1. Compare the results and comment on any differences you see.