**NATIONAL UNIVERSITY OF SINGAPORE**
**Department of Mathematics**
**Semester II (2007/2008) MA4254 Discrete Optimization Tutorial 9**

Solution or hint to selected questions

**Q2.** (i) The LP relaxation of the $0-1$ knapsack problem can be written as

$$\begin{array}{ll} \max & c^T x \\ \text{s.t.} & a^T x \le b \\ & -Ix \le 0 \\ & Ix \le \bar{e}, \end{array}$$

where $\bar{e}$ is the $n$-vector of all ones. Then its dual is

$$\begin{array}{ll} \min & by + 0^T u + \bar{e}^T v \\ \text{s.t.} & [a \ \ -I \ \ I] \begin{bmatrix} y \\ u \\ v \end{bmatrix} = c \\ & y \ge 0, u \ge 0, v \ge 0\,. \end{array}$$

(ii) Firstly, $x^*$ is a feasible solution to the LP relaxation. Secondly, the coefficient matrix of active constrains

$$B^T := \begin{bmatrix} a^T \\ -e_{r+2} \\ \vdots \\ -e_n \\ e_1 \\ \vdots \\ e_r \end{bmatrix}$$

is nonsingular. Finally, by the definition, $x^*$ is a basic feasible solution.

(iii) Let $u_i^* = 0$ for $i = 1, \ldots, r+1$ and $v_j^* = 0$ for $j = r+1, \ldots, n$. Then from

$$ay^* - u^* + v^* = c$$

we have

$$y^* = c_{r+1}/a_{r+1},$$

$$u_i^* = a_i \times c_{r+1}/a_{r+1} - c_i \ \ , i = r+2, \ldots, n,$$

$$v_j^* = c_j - a_j \times c_{r+1}/a_{r+1} \ \ , j = 1, \ldots, r.$$

Because the ratios $c_i/a_i$ are computed and ordered from large to small, $(y^*, u^*, v^*)$ is nonnegative. By the nonsingularity of $B$, we know that $(y^*, u^*, v^*)$ is a basic feasible solution to the dual problem obtained in part (i)

(iv) Since $c^T x^* = y^* b + \bar{e}^T v^*$, by duality we know that $x^*$ solves the LP relaxation.