PARITIES OF v-DECOMPOSITION NUMBERS

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Abstract. We prove that the v-decomposition number $d_{\lambda\mu}(v)$ arising from the canonical basis of the Fock space representation of $U_v(\widehat{\mathfrak{sl}}_e)$ is an even or odd polynomial according to whether the partitions λ and μ have the same relative e-sign (or e-parity) or not.

1. Introduction

Let v be an indeterminate, and let e be an integer greater than 1. The Fock space representation $\mathcal{F}$ of $U_v(\widehat{\mathfrak{sl}}_e)$, as a $\mathbb{C}(v)$-vector space, has two distinguished bases, the standard basis $\{s(\lambda) \mid \lambda \in \mathcal{P}\}$ and the canonical basis $\{G(\lambda) \mid \lambda \in \mathcal{P}\}$, both being indexed by the set $\mathcal{P}$ of all partitions of non-negative integers. The v-decomposition number $d_{\lambda\mu}(v) \in \mathbb{C}(v)$ is the coefficient of $s(\lambda)$ when the canonical basis element $G(\mu)$ is expressed in terms of the standard basis elements, i.e.

$$G(\mu) = \sum_{\lambda \in \mathcal{P}} d_{\lambda\mu}(v) s(\lambda).$$

Varagnolo and Vasserot [14] showed that the v-decomposition numbers are parabolic Kazhdan-Lusztig's polynomials. As an immediate consequence, a non-zero v-decomposition number is a sum of monomials in v, all of which are either of odd degree or of even degree. In this paper, we provide a combinatorial criterion, in terms of relative e-signs (or e-parities) of the partitions λ and μ, that determines exactly which of the two possibilities occurs. As a corollary, we relate the relative e-sign of $m(\lambda)$ to that of λ when λ is e-regular; here, $\lambda \mapsto m(\lambda)$ is the Mullineux's involution.

Our results prove to be particularly useful in the study of blocks of small weight of symmetric groups and related algebras; please see [11, 12] for some applications of these results.

Parities have always played an important role in the representation theory of symmetric groups and general linear groups. For example, Lusztig's conjecture for general linear groups is known to be equivalent to its Ext-groups satisfying some parity condition (see [2]). Lately, Parshall and Scott [10] showed that the existence of a $(\mathbb{Z}/2\mathbb{Z})$-based Kazhdan-Lusztig theory for the module category will imply the truth of Lusztig's conjecture. For symmetric groups, it has been observed that many blocks of symmetric groups

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Date: April 2008.

2000 *Mathematics Subject Classification.* 17B37.

The author, supported by Academic Research Fund R-146-000-043-112 of National University of Singapore, thanks Bernard Leclerc for his suggestion of the proof of Theorem 2.4, and Hung Yean Loke for directing him to [5] in which the formula for the length of an element of a general affine Weyl group can be found.

have bipartite Ext-quivers, giving important information about the radical series of its projective indecomposable modules, and it appears that this should hold for any such block with an Abelian defect group. More recently, Fayers and the author [4] showed that the Ext-quiver of every weight 3 block of symmetric group with Abelian defect group is bipartite, with the bipartition being described very simply by the relative p-signs of partitions labelling the simple modules (where p is the characteristic of the underlying field). One may then conjecture that whenever a block of the symmetric group has a bipartite Ext-quiver, then the bipartition should be similarly described by the relative p-signs of partitions labelling the simple modules.

By the results of Varagnolo-Vasserot [14] and Ariki [1], upon evaluation at $v = 1$, the v-decomposition numbers give the decomposition numbers of q-Schur algebras and Iwahori-Hecke algebras in characteristic zero, which are deformations of the Schur algebras and symmetric group algebras respectively. This suggests that the v-decomposition numbers describe some filtration of the Weyl modules of q-Schur algebras, or the Specht modules of the Iwahori-Hecke algebras. Our result provides some evidence that the filtration concerned is the radical filtration. This is consistent with James's conjecture (which, informally speaking, asserts that symmetric group blocks with Abelian defect groups and their corresponding blocks of Schur algebras behave much in the same way as the corresponding blocks of Iwahori-Hecke algebras and q-Schur algebras in characteristic zero) and the conjecture that the Ext-quivers of these blocks are bipartite, with the bipartition described by the relative e-signs of partitions labelling the simple modules. We note that this does not refute an earlier conjecture of Lascoux, Leclerc, Thibon and Rouquier [6] that the filtration concerned is the Jantzen filtration, as many experts believe that the Jantzen filtration and the radical filtration coincide for these blocks.

We now indicate the layout of this paper. In the remainder of this section, we give a summary of the combinatorics of partitions which we shall require. In Section 2, we review the theory of v-decomposition numbers, state the main result of this paper and prove it while assuming Proposition 2.3. Section 3 is devoted entirely to the proof of Proposition 2.3.

1.1. **Partitions.** A partition $\lambda = (\lambda_1, \lambda_2, \dots)$ is a weakly decreasing sequence of non-negative integers, where for sufficiently large i, $\lambda_i = 0$. If $\sum_i \lambda_i = n$, we say λ is a partition of n. The length of λ, denoted $l(\lambda)$, equals $\max(i \mid \lambda_i > 0)$. Denote the set of partitions of n by $\mathcal{P}_n$, and let $\mathcal{P} = \bigcup_n \mathcal{P}_n$ be the set of all partitions.

A strictly decreasing sequence $\beta = (\beta_1, \beta_2, \dots, \beta_s)$ of non-negative integers is a sequence of β-numbers for λ if $s \geq l(\lambda)$, and $\beta_i = \lambda_i + s - i$ for all $1 \leq i \leq s$. Every strictly decreasing sequence of non-negative integers is a sequence of β-numbers for a unique partition.

The James e-abacus has e vertical runners, labelled $0, 1, \dots, e-1$. Its positions are labelled from left to right, and top down, starting from 0. The partition λ may be displayed on the abacus as follows: if $\beta = (\beta_1, \beta_2, \dots, \beta_s)$ is a sequence of β-numbers for λ, then we place a bead at position β_i for each i. This is the (e-)abacus display of λ with s beads.

In an abacus display of λ, moving a bead from position a to a vacant position b, with $a > b$, corresponds to removing a (rim) hook of length $a-b$ from λ. The number of beads crossed in so doing (i.e. the number of occupied positions between b and a) is the leg-length of the hook. The e-core of λ is thus obtained when we slide the beads as far up their respective runners as possible. The e-weight of λ is the total number of times we slide the beads one position up their respective runners to obtain its e-core. The relative (e-)sign of λ, denoted as $\sigma_e(\lambda)$, can be defined as $(-1)^t$, where t is the total number of beads crossed to obtain the e-core (see [8, §2]).

The conjugate partition of λ, denoted $\lambda' = (\lambda'_1, \lambda'_2, \dots)$, is defined by $\lambda'_j = |\{i \mid \lambda_i \geq j\}|$ for all $j \in \mathbb{Z}^+$. Given an abacus display of λ, we can obtain the abacus display of λ' by rotating the abacus of λ through an angle of π, and read the vacant positions as occupied and the occupied positions as vacant. Thus, λ' has the same e-weight as λ, and its e-core is the conjugate partition of the e-core of λ.

The partition λ is e-regular if there does not exist i such that $\lambda_i = \lambda_{i+1} = \cdots = \lambda_{i+e-1} > 0$. In [9], Mullineux formulated an involution $\lambda \mapsto m(\lambda)$ on the set of e-regular partitions of $\mathcal{P}_n$, now commonly known as Mullineux's map. We refer the reader to [9] for a combinatorial description of this involution.

2. The Fock space representation of $U_v(\widehat{\mathfrak{sl}}_e)$

In this section, we discuss briefly some of the remarkable properties enjoyed by the v-decomposition numbers arising from the Fock space representation of $U_v(\widehat{\mathfrak{sl}}_e)$.

The Fock space representation $\mathcal{F}$ of $U_v(\widehat{\mathfrak{sl}}_e)$, as a $\mathbb{C}(v)$-vector space, has two distinguished bases: the standard basis $\{s(\lambda) \mid \lambda \in \mathcal{P}\}$ and the canonical basis $\{G(\lambda) \mid \lambda \in \mathcal{P}\}$. The v-decomposition number $d_{\lambda\mu}(v) \in \mathbb{C}(v)$ is the coefficient of $s(\lambda)$ when the canonical basis element $G(\mu)$ is expressed in terms of the standard basis elements, i.e.

$$G(\mu) = \sum_{\lambda \in \mathcal{P}} d_{\lambda\mu}(v) s(\lambda).$$

It is known that $d_{\lambda\mu}(v) \neq 0$ only if λ and μ have the same e-weight and e-core (see, for example, [7, Proposition 11]).

Varagnolo and Vasserot [14] showed that the v-decomposition numbers are parabolic Kazhdan-Lusztig's polynomials. We give a brief account of this.

Let $\mathfrak{S}_n$ be the symmetric group on n letters. The extended affine Weyl group $W = \mathfrak{S}_n \ltimes \mathbb{Z}^n$ acts on $\mathbb{Z}^n$ via

$$\sigma(t_1, \dots, t_n) \cdot (a_1, \dots, a_n) = (t_{\sigma^{-1}(1)} + a_{\sigma^{-1}(1)}, \dots, t_{\sigma^{-1}(n)} + a_{\sigma^{-1}(n)}).$$

The set $\mathcal{A} = \{(x_1, \dots, x_n) \in \mathbb{Z}^n \mid -e < x_1 \leq \cdots \leq x_n \leq 0\}$ is a fundamental domain of this action, and $\{\varepsilon_i - \varepsilon_j \mid i \neq j\}$ is a root system for W, where ε_i is the i-th standard basis element of $\mathbb{Z}^n$. We take the positive roots to be $\{\varepsilon_i - \varepsilon_j \mid i < j\}$. For $a \in \mathbb{Z}^n$, write w_a for the (unique) element in $\mathfrak{S}_n \ltimes \mathbb{Z}^n$ having the minimal length (with respect to this positive root system) such that $w_a^{-1} \cdot a \in \mathcal{A}$.

Given a partition λ with $l(\lambda) \leq n$, write $\widehat{\lambda}$ for the strictly increasing sequence of non-negative integers $(a_1, \dots, a_n)$ such that $(a_n, \dots, a_1)$ is a sequence of β-numbers of λ. Let μ be another partition with $l(\mu) \leq n$, and assume λ and μ have the same e-weight and e-core. Then $\widehat{\lambda}$ and $\widehat{\mu}$ lie in the same W-orbit, which intersects $\mathcal{A}$ at α, say. Let w^α be the longest element in the stabilizer of α (under the action of W). We have

Theorem 2.1 ([14]; see also [7, Theorem 13])**.**

$$d_{\lambda\mu}(v) = \sum_{y \in \mathfrak{S}_r} (-v)^{\ell(y)} P_{y w_{\widehat{\lambda}} w^\alpha, w_{\widehat{\mu}} w^\alpha}(v),$$

where $\ell(y)$ is the length of y as an element of W (with respect to the chosen positive root system), and $P_{x,w}$ is the coefficient of T_x in the expansion of the Kazhdan-Lusztig base element C'_w of the Hecke algebra associated with W (following Soergel's convention on the normalisation of the generators T_i [13]*).*

The upshot of this is:

Corollary 2.2. *Keeping the above notations, we have*

$$d_{\lambda\mu}(v) \in \begin{cases} \mathbb{N}_0[v^2], & \text{if } (-1)^{\ell(w_{\widehat{\lambda}})} = (-1)^{\ell(w_{\widehat{\mu}})}; \\ v\mathbb{N}_0[v^2], & \text{if } (-1)^{\ell(w_{\widehat{\lambda}})} \neq (-1)^{\ell(w_{\widehat{\mu}})}. \end{cases}$$

Proof. This follows from the fact that

$$P_{x,w}(v) \in \begin{cases} \mathbb{N}_0[v^2], & \text{if } (-1)^{\ell(w)} = (-1)^{\ell(x)}; \\ v\mathbb{N}_0[v^2], & \text{if } (-1)^{\ell(w)} \neq (-1)^{\ell(x)}. \end{cases}$$

□

In the next section, we relate $\ell(w_{\widehat{\lambda}})$ with the e-weight and the relative e-sign of λ, and $\ell(w_{\widehat{\kappa}})$, where κ is the e-core of λ. More specifically, we prove the following:

Proposition 2.3. *Let λ be a partition with e-weight ϖ and e-core κ, and assume that $l(\lambda) \leq n$. Then*

$$(-1)^{\ell(w_{\widehat{\lambda}})} = (-1)^{\varpi(n-1) + \ell(w_{\widehat{\kappa}})} \sigma_e(\lambda).$$

The following theorem immediately follows from Corollary 2.2 and Proposition 2.3:

Theorem 2.4. *If $d_{\lambda\mu}(v) \neq 0$, then*

$$d_{\lambda\mu}(v) \in \begin{cases} \mathbb{N}_0[v^2], & \text{if } \sigma_e(\lambda) = \sigma_e(\mu); \\ v\mathbb{N}_0[v^2], & \text{if } \sigma_e(\lambda) \neq \sigma_e(\mu). \end{cases}$$

We conclude this section with a result relating the relative sign of an e-regular partition λ with that of $m(\lambda)$. It is proved in [4, Proposition 2.19] that when p is an odd prime and λ is a p-regular partition with p-weight 3, $\sigma_p(m(\lambda)) \neq \sigma_p(\lambda)$. Furthermore, it is remarked that $\sigma_p(m(\lambda)) = (-1)^w \sigma_p(\lambda)$ holds in general when λ has p-weight w (and is p-regular), by using v-decomposition numbers, without giving details. Here we use Theorem 2.4 to provide a formal proof of a more general version of this statement.

Proposition 2.5. *Suppose λ is a e-regular partition having e-weight ϖ. Then $\sigma_e(m(\lambda)) = (-1)^{e\varpi}\sigma_e(\lambda)$.*

Proof. Since $d_{m(\lambda)'\lambda}(v) = v^{\varpi}$ [6, Corollary 7.7], we have $\sigma_e(m(\lambda)')\sigma_e(\lambda) = (-1)^{\varpi}$ by Theorem 2.4. Note that $\sigma_e(\mu') = (-1)^{(e-1)\varpi}\sigma_e(\mu)$ for a e-regular partition μ with e-weight ϖ. Thus, $\sigma_e(m(\lambda)) = (-1)^{e\varpi}\sigma_e(\lambda)$. □

3. The extended affine Weyl group $W = \mathfrak{S}_n \ltimes \mathbb{Z}^n$ action on $\mathbb{Z}^n$

We provide a proof of Proposition 2.3 in this section. Our goal is to relate the (parity of the) length of $w_{\widehat{\lambda}}$ to that of $w_{\widehat{\kappa}}$, where κ is the e-core of λ.

Recall the set of positive roots of W, and the fundamental domain $\mathcal{A}$ of the action of W on $\mathbb{Z}^n$, as described in the previous section.

For $T = \sum_{i=1}^n T_i\varepsilon_i \in \mathbb{Z}^n$, and $\sigma \in \mathfrak{S}_n$, the length of $T\sigma$ as an element of W may be calculated based on the following formula, which is a specialization of that for general extended affine Weyl groups found in Proposition 1.23 of [5]:

Theorem 3.1.
$$\ell(T\sigma) = \sum_{\substack{i<j \\ \sigma^{-1}(i)<\sigma^{-1}(j)}} |T_i - T_j| + \sum_{\substack{i<j \\ \sigma^{-1}(i)>\sigma^{-1}(j)}} |T_i - T_j - 1|.$$

Observe that the formula agrees with that for the ordinary Weyl group $\mathfrak{S}_n$ upon restriction.

Fix an element $a = \sum_{i=1} a_i\varepsilon_i \in \mathbb{Z}^n$, and for each $i = 1, \dots, n$, let $t_i = \lceil a_i/e \rceil$ so that $-e < a_i - et_i \le 0$. Write $t = \sum_{i=1} t_i\varepsilon_i$ and $c = \sum_{i=1} c_i\varepsilon_i = -t \cdot a$. Note that $w^{-1} \cdot a \in \mathcal{A}$ for $w \in \mathfrak{S}_n \ltimes \mathbb{Z}^n$ if and only if $w = t\sigma$ for some $\sigma \in \mathfrak{S}_n$ with $\sigma^{-1} \cdot c \in \mathcal{A}$.

A description of w_a is particularly easy when $a_1 \le a_2 \le \cdots \le a_n$.

Proposition 3.2. *If $T = \sum_{i=1}^n T_i\varepsilon_i \in \mathbb{Z}^n$ with $T_1 \le T_2 \le \cdots \le T_n$, and $\sigma \in \mathfrak{S}_n$, then*
$$\begin{aligned}\ell(T\sigma) &= \sum_{i<j}(T_j - T_i) + \sum_{\substack{i<j \\ \sigma^{-1}(i)>\sigma^{-1}(j)}} 1 \\ &= \sum_{i<j}(T_j - T_i) + \ell(\sigma).\end{aligned}$$

In particular, if a, t and c are as defined above, with $a_1 \le a_2 \le \cdots \le a_n$, then $w_a = t\sigma_c$, where σ_c is shortest element of $\mathfrak{S}_n$ satisfying $\sigma_c^{-1} \cdot c \in \mathcal{A}$, and
$$\begin{aligned}\ell(w_a) &= \sum_{i<j}(t_j - t_i) + l(\sigma_c) \\ &= \sum_{i<j}(t_j - t_i) + |\{(i,j) \mid 1 \le i < j \le n,\ c_i > c_j\}|.\end{aligned}$$

Proof. The formula for $\ell(T\sigma)$ follows immediately from Theorem 3.1. If $a_1 \le a_2 \le \cdots \le a_n$, then necessarily $t_1 \le t_2 \le \cdots \le t_n$, so that $w_a = t\sigma_c$ where σ_c is the shortest element of $\mathfrak{S}_n$ satisfying $\sigma_c^{-1} \cdot c \in \mathcal{A}$. The description of $\ell(w_a)$ then follows. □

Now, assume $a_1 < a_2 < \cdots < a_n$, and suppose $a_{s-1} < a_r - e < a_s$ for some $1 \le s \le r \le n$. Let u be the least index such that $t_u = t_r$. Then $s \le u \le r$. Furthermore,

Lemma 3.3.

(1) $t_u = t_{u+1} = \cdots = t_r$, *and if* $s < u$, *then* $t_s = t_{s+1} = \cdots = t_{u-1} = t_r - 1$.

(2) $c_u < c_{u+1} < \cdots < c_r$, *and if* $s < u$, *then* $c_r < c_s < c_{s+1} < \cdots < c_{u-1}$.

Proof. Since $a_r - e < a_i < a_r$ for all $s \le i \le r-1$, we have $t_r - 1 \le t_i \le t_r$. This yields part (1). Part (2) then follows since $c_i = a_i - et_i$ for all i (note that if $s < u$, then $c_r = a_r - et_r < a_s + e - e(t_s + 1) = c_s$). □

Write $[s, r]$ for $(s, s+1, \ldots, r) \in \mathfrak{S}_n$, and let

$$\begin{aligned} a' = (a'_1, \ldots, a'_n) &= [s, r](-e\varepsilon_r) \cdot a \\ &= (a_1, \ldots, a_{s-1}, a_r - e, a_s, \ldots, a_{r-1}, a_{r+1}, \ldots, a_n) \end{aligned}$$

For each i, let $t'_i = \lceil a'_i / e \rceil$ and $c'_i = a'_i - et'_i$, and write $t' = (t'_1, \ldots, t'_n)$ and $c' = (c'_1, \ldots, c'_n)$. Then $c' = [s, r] \cdot c$, and

Lemma 3.4. $t' = t - \varepsilon_u$.

Proof. This follows from part (1) of Lemma 3.3. □

We wish to compare $\ell(w_a)$ and $\ell(w_{a'})$. First, we compare $\ell(\sigma_c)$ and $\ell(\sigma_{c'})$.

Lemma 3.5. $\ell(\sigma_c) - \ell(\sigma_{c'}) = 2u - (r + s)$.

Proof. We define a partial correspondence between the sets $X = \{(i, j) \mid i < j,\ c_i > c_j\}$ and $Y = \{(i, j) \mid i < j,\ c'_i > c'_j\}$ as follows:

$$\begin{aligned} (i, j) &\longleftrightarrow (i^+, j^+) \qquad (j \ne r) \\ (i, r) &\longleftrightarrow (i, s) \qquad\quad (i < s), \end{aligned}$$

where

$$i^+ = \begin{cases} i + 1, & \text{if } s \le i < r; \\ i, & \text{otherwise.} \end{cases}$$

By part (2) of Lemma 3.3, this is actually a one-to-one correspondence between the set $X \setminus X'$ and $Y \setminus Y'$, where

$$\begin{aligned} X' &= \{(i, r) \mid s \le i < u\}, \\ Y' &= \{(s, j) \mid u < j \le r\}. \end{aligned}$$

Thus, $\ell(\sigma_c) - \ell(\sigma'_c) = |X| - |Y| = |X'| - |Y'| = (u - s) - (r - u) = 2u - (r + s)$. □

Corollary 3.6. $\ell(w_a) = \ell(w_{a'}) + 4u - n - 1 - r - s$.

Proof. By Proposition 3.2, Lemmas 3.4 and 3.5, we have

$$\begin{aligned} \ell(w_a) &= \sum_{i<j} (t_j - t_i) + \ell(\sigma_c) \\ &= \sum_{i<j} (t'_j - t'_i) + 2u - n - 1 + \ell(\sigma_{c'}) + 2u - r - s \\ &= \ell(w_{a'}) + 4u - n - 1 - r - s. \end{aligned}$$

□

We are now ready to prove Proposition 2.3.

Proof of Proposition 2.3. We prove by induction on the e-weight ϖ of λ, with $\varpi = 0$ being trivial. Let λ have positive e-weight ϖ, and let μ be the partition obtained when one particular e-hook is removed from λ. Let $\widehat{\lambda} = (a_1, \dots, a_n)$. Then there exist some integers r, s with $1 \leq s \leq r \leq n$ such that $\widehat{\mu} = [s, r](-e\varepsilon_r) \cdot \widehat{\lambda}$. Thus,

$$\begin{aligned}(-1)^{\ell(w_{\widehat{\lambda}})} &= (-1)^{\ell(w_{\widehat{\mu}})}(-1)^{(n-1)+(r-s)} \\ &= (-1)^{\ell(w_{\widehat{\kappa}})+(\varpi-1)(n-1)}\sigma_e(\mu)(-1)^{(n-1)+(r-s)} \\ &= (-1)^{\ell(w_{\widehat{\kappa}})+\varpi(n-1)}\sigma_e(\lambda)\end{aligned}$$

by Corollary 3.6, and induction hypothesis (note that the e-hook removed from λ to obtain μ has leg-length $r - s$). □

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