Abstract

In the teaching of a first undergraduate course in Linear Algebra, the basic concepts of matrices and linear systems are covered. When students are solving linear algebra problems, it is very likely that they will need to perform tedious computation like matrix multiplication and Gaussian elimination. Most graphing calculators (GC) are capable of performing these computations. However, not all undergraduate students (in my university) are equipped with GC, and the common practice is that students are only allowed to use scientific calculators for their examinations. During a recent semester, I allowed a special group of students to use GC and restricted the rest of the students to use scientific calculators during the same examination. I studied and compared the examination results of the two groups of students. In this talk, I will present my findings.
Overview

- **Usage of GC in Singapore Schools**
  - Pre-University
  - University

- **Linear Algebra Course in NUS**
  - Course description
  - Students’ profile
  - Assessments

- **Students’ Feedback**
  - Survey
  - Exam results

- **Concluding Remarks**
Usage of GC in Singapore Schools

Pre-University (JC) Level
- Introduced to all JC students in the first year
- Used during GCE A level math exam
- Most JC’s use TI-84 Plus
Usage of GC in Singapore Schools

University Level
- Not commonly used in math classes
- Students from non-JC background
- University lecturers are not familiar with GC
- Concept VS computation
- Alternative software available

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GC for Linear Algebra Examination
Linear Algebra Course in NUS

- First year undergraduate course
- Offered in both semester 1 and 2
- 300 ~ 400 students
- Mainly for math major and science students
- Topics:
Linear Algebra Course in NUS

Students’ profile (Semester 2)

- Undergraduate students
  - Math Major
  - Non-Math Science students
  - Non-Science students
- H3 students
  - Current JC year 2 students
  - Selected from JC students based on their math results
Linear Algebra Course in NUS

Assessment
- Practice Sessions: Any calculators
- Lab Sessions: MATLAB
- Mid-Term Exam: Only scientific calculators (traditionally)
- Final Exam: Any calculators
Questions

- Should students be allowed to use GC during exam?
- Will students who use GC during exam have an advantage over students who don’t?
- Is it possible to set an exam that is fair to all students with or without using GC?
Linear Algebra Course in NUS

Assessment
- Practice Sessions: Any calculators
- Lab Sessions: MATLAB
- Mid-Term Exam: Only scientific calculators (traditionally)
- Final Exam: Undergrad: only scientific calculators, H3 students: graphing calculators are allowed

Undergrad: only scientific calculators
H3 students: graphing calculators are allowed
Usage of GC in Linear Algebra

- Reduced row echelon form
- Matrix multiplication
- Determinant
- Inverse matrix
- Elementary row operations
Students’ Feedback

Post mid-term exam survey

75 H3 students

69 brought the GC
42 used the GC
27 did not use the GC

6 did not bring GC
27 did not use the GC
Post mid-term exam survey

Question 1 (a) [5 marks]

The row echelon form of the augmented matrix for a linear system is given by

\[
\begin{pmatrix}
1 & 1 & -1 & 1 & 0 & 0 \\
0 & 1 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 2 & 0 & 6
\end{pmatrix}
\]

(i) Write down the general solution of the linear system.
(Use \(x_1, x_2, x_3, x_4, x_5\) for the variables of the system.)

(ii) Write down a specific solution of the linear system with \(x_1 = 0\) and \(x_2 = 0\).

Number of students using GC for this part: 21

Number of students who think GC (but not SC) can help for this part: 22


**Students’ Feedback**

**Post mid-term exam survey**

**Question 1 (b) [5 marks]**

A linear system is given by

\[
\begin{cases}
    ax + az &= a \\
    ax + ay + 2az &= 1 + a
\end{cases}
\]

(i) Find all values of \( a \) such that the system has no solution.

(ii) Find all values of \( a \) such that the system has infinitely many solutions.

(iii) Suppose a third equation \( ax + ay + az = b \) is added to the above system. Find all possible values of \( a \) and \( b \) such that the new system has exactly one solution.

Number of students using GC for this part: 2

Number of students who think GC (but not SC) can help for this part: 3
Students’ Feedback

Post mid-term exam survey

Question 2 (a) [6 marks]
Let $A$ and $B$ be two $3 \times 3$ matrices such that

$$A \xrightarrow{R_1 + 5R_3} \xrightarrow{R_3 \rightarrow R_2} \xrightarrow{4R_2} B.$$ 

(i) Write down the three elementary matrices $E_1$, $E_2$, $E_3$ such that

$$E_3E_2E_1A = B.$$ 

(ii) Find a matrix $C$ such that $A = CB$.

Number of students using GC for this part: 6

Number of students who think GC (but not SC) can help for this part: 9
Students’ Feedback

Post mid-term exam survey

Question 2 (b) [4 marks]
Let $A$ and $B$ be as in part (a) and $b$ be a $3 \times 1$ column matrix. Suppose the linear system $Ax = b$ has no solution. How many solutions does $Bx = b$ have? Justify your answer.

Number of students using GC for this part: 4
Number of students who think GC (but not SC) can help for this part: 6
Post mid-term exam survey

Question 3 (a) [6 marks]
Let \( A = \begin{pmatrix} a & b & a+b \\ b & a & a+b \\ 0 & a+b & a+b \end{pmatrix} \).

(i) Write down the matrix \( B \) by adding first column of \( A \) to second column.
(ii) Use part (i) to find the determinant of \( A \). Explain briefly how you get the answer.
(iii) Find the values of \( a \) and \( b \) for \( A \) to be an invertible matrix. Justify your answer.

Number of students using GC for this part: 6
Number of students who think GC (but not SC) can help for this part: 14
Students’ Feedback

Post mid-term exam survey

Question 3 (b) [4 marks]
Suppose $adj\ C$ is invertible. Show that $C$ is invertible.

Number of students using GC for this part: 0
Number of students who think GC (but not SC) can help for this part: 5
Students’ Feedback

Post mid-term exam survey

Question 4 (a) [5 marks]
Let \( W = \{ (x, y, z) \mid x + y - z = 0 \} \) be a subset of \( \mathbb{R}^3 \).

(i) Express \( W \) in explicit set notation.

(ii) Express \( W \) as a linear span (if possible).

(iii) Is \( W \) a subspace of \( \mathbb{R}^3 \)? Justify your answer.

Number of students using GC for this part: 2
Number of students who think GC (but not SC) can help for this part: 9
Students’ Feedback

Post mid-term exam survey

Question 4 (b) [5 marks]
Let \( v_1 = (1, 1, -1), v_2 = (-1, 1, 1) \) and \( u_1 = (0, 1, 0), u_2 = (1, 0, -1), u_3 = (1, 1, 1) \).

(i) Show that \( \text{span}\{v_1, v_2\} \subseteq \text{span}\{u_1, u_2, u_3\} \).

(ii) Is \( \text{span}\{v_1, v_2\} = \text{span}\{u_1, u_2, u_3\} \)? Justify your answer.

Number of students using GC for this part: 14
Number of students who think GC (but not SC) can help for this part: 14
Post mid-term exam survey

<table>
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<th>GC has improved your performance in this exam.</th>
<th>No. of students</th>
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<tr>
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<tr>
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</tr>
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### Students’ Feedback

#### Post mid-term exam survey

Without GC, you will perform equally well in this exam

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<tr>
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<td>7</td>
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<td>18</td>
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</table>
Post mid-term exam survey

<table>
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<tr>
<th>Do you think that GC is necessary for this exam?</th>
<th>No. of students</th>
</tr>
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<tbody>
<tr>
<td>Yes</td>
<td>15</td>
</tr>
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<td>No</td>
<td>44</td>
</tr>
<tr>
<td>Not sure</td>
<td>14</td>
</tr>
</tbody>
</table>
Students’ Feedback

- It takes a long time to key in matrices in GC
- Can’t use GC for questions with unknowns
- Manual calculation can be faster for small matrices
- GC will not improve one’s performance as workings still need to be shown
- GC may discourage us to practice the Gaussian Elimination
- SC is equally capable (as GC)
- Do not know how to use GC for matrix calculation
- No time to use
- Save time in performing Gaussian Elimination
- GC is a good tool for error checking (23)
Concluding Remarks

Questions

- Should students be allowed to use GC during exam?
  - They can be allowed to use GC

- Will students who use GC during exam have an advantage over students who don’t?
  - Not significant

- Is it possible to set an exam that is fair to all students with or without using GC?
  - Yes
Concluding Remarks

Future improvement of the study
- Post final exam survey
- Comparison between H3 and undergrad
- Comparing the performance of H3 students with and without using GC
- Allowing undergrad to use GC in exam
- Prepare the H3 students on the use of GC

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