

Using Graphing Calculators for Linear Algebra Examination

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Abstract

In the teaching of a first undergraduate course in Linear Algebra, the basic concepts of matrices and linear systems are covered. When students are solving linear algebra problems, it is very likely that they will need to perform tedious computation like matrix multiplication and Gaussian elimination. Most graphing calculators (GC) are capable of performing these computations. However, not all undergraduate students (in my university) are equipped with GC, and the common practice is that students are only allowed to use scientific calculators for their examinations. During a recent semester, I allowed a special group of students to use GC and restricted the rest of the students to use scientific calculators during the same examination. I studied and compared the examination results of the two groups of students. In this talk, I will present my findings.

Overview

- Usage of GC in Singapore Schools
 - Pre-University
 - University
- Linear Algebra Course in NUS
 - Course description
 - Students' profile
 - Assessments
- Students' Feedback
 - Survey
 - Exam results
- Concluding Remarks

Usage of GC in Singapore Schools

Pre-University (JC) Level

- Introduced to all JC students in the first year
- Used during GCE A level math exam
- Most JC's use TI-84 Plus

Usage of GC in Singapore Schools

University Level

- Not commonly used in math classes
- Students from non-JC background
- University lecturers are not familiar with GC
- Concept VS computation
- Alternative software available

Linear Algebra Course in NUS

- First year undergraduate course
- Offered in both semester 1 and 2
- 300 ~ 400 students
- Mainly for math major and science students
- Topics:
Systems of linear equations. Matrices. Determinants. Euclidean n -space. Subspaces. Linear independence. Basis and dimension. Rank of a matrix. Orthogonality and orthonormal bases. Eigenvalues and eigenvectors. Diagonalization. Linear transformations.

Linear Algebra Course in NUS

Students' profile (Semester 2)

- Undergraduate students
 - Math Major
 - Non-Math Science students
 - Non-Science students
- H3 students
 - Current JC year 2 students
 - Selected from JC students based on their math results

Linear Algebra Course in NUS

Assessment

- Practice Sessions Any calculators
- Lab Sessions MATLAB
- Mid-Term Exam Only scientific calculators
- Final Exam (traditionally)

Linear Algebra Course in NUS

Questions

- Should students be allowed to use GC during exam?
- Will students who use GC during exam have an advantage over students who don't?
- Is it possible to set an exam that is fair to all students with or without using GC?

Linear Algebra Course in NUS

Assessment

- Practice Sessions Any calculators
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- Mid-Term Exam Only scientific calculators
- Final Exam (traditionally)



Undergrad: **only scientific calculators**

H3 students: **graphing calculators are allowed**

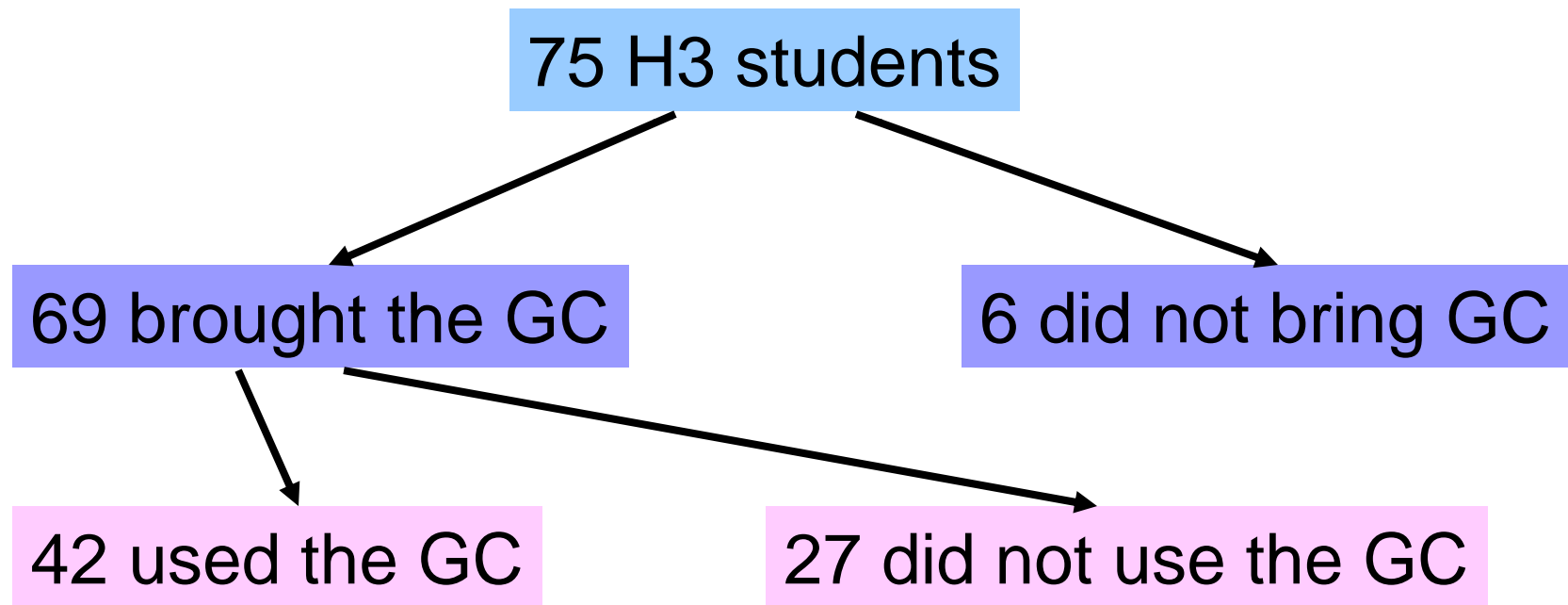
Linear Algebra Course in NUS

Usage of GC in Linear Algebra

- Reduced row echelon form
- Matrix multiplication
- Determinant
- Inverse matrix
- Elementary row operations

Students' Feedback

Post mid-term exam survey



Students' Feedback

Post mid-term exam survey

Question 1 (a) [5 marks]

The row echelon form of the augmented matrix for a linear system is given by

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 6 \end{array} \right)$$

- (i) Write down the general solution of the linear system.
(Use x_1, x_2, x_3, x_4, x_5 for the variables of the system.)
- (ii) Write down a specific solution of the linear system with $x_1 = 0$ and $x_2 = 0$.

Number of students using GC for this part: 21

Number of students who think GC (but not SC) can help for this part: 22

Students' Feedback

Post mid-term exam survey

Question 1 (b) [5 marks]

A linear system is given by

$$\begin{cases} ax & + & az & = & a \\ ax & + & ay & + & 2az & = & 1 + a \end{cases}$$

- (i) Find all values of a such that the system has no solution.
- (ii) Find all values of a such that the system has infinitely many solutions.
- (iii) Suppose a third equation $ax + ay + az = b$ is added to the above system. Find all possible values of a and b such that the new system has exactly one solution.

Number of students using GC for this part: 2

Number of students who think GC (but not SC) can help for this part: 3

Students' Feedback

Post mid-term exam survey

Question 2 (a) [6 marks]

Let A and B be two 3×3 matrices such that

$$A \begin{array}{l} R_1 + 5R_3 \\ \rightarrow \end{array} \begin{array}{l} R_3 \leftrightarrow R_2 \\ \rightarrow \end{array} \begin{array}{l} 4R_2 \\ \rightarrow \end{array} B.$$

(i) Write down the three elementary matrices E_1, E_2, E_3 such that

$$E_3 E_2 E_1 A = B.$$

(ii) Find a matrix C such that $A = CB$.

Number of students using GC for this part: 6

Number of students who think GC (but not SC) can help for this part: 9

Students' Feedback

Post mid-term exam survey

Question 2 (b) [4 marks]

Let A and B be as in part (a) and b be a 3×1 column matrix. Suppose the linear system $Ax = b$ has no solution. How many solutions does $Bx = b$ have? Justify your answer.

Number of students using GC for this part: 4

Number of students who think GC (but not SC) can help for this part: 6

Students' Feedback

Post mid-term exam survey

Question 3 (a) [6 marks]

$$\text{Let } A = \begin{pmatrix} a & b & a+b \\ b & a & a+b \\ 0 & a+b & a+b \end{pmatrix}.$$

- (i) Write down the matrix B by adding first column of A to second column.
- (ii) Use part (i) to find the determinant of A . Explain briefly how you get the answer.
- (iii) Find the values of a and b for A to be an invertible matrix. Justify your answer.

Number of students using GC for this part: 6

Number of students who think GC (but not SC) can help for this part: 14

Students' Feedback

Post mid-term exam survey

Question 3 (b) [4 marks]

Suppose $\text{adj } C$ is invertible. Show that C is invertible.

Number of students using GC for this part: 0

Number of students who think GC (but not SC) can help for this part: 5

Students' Feedback

Post mid-term exam survey

Question 4 (a) [5 marks]

Let $W = \{(x, y, z) \mid x + y - z = 0\}$ be a subset of \mathbb{R}^3 .

- (i) Express W in explicit set notation.
- (ii) Express W as a linear span (if possible).
- (iii) Is W a subspace of \mathbb{R}^3 ? Justify your answer.

Number of students using GC for this part: 2

Number of students who think GC (but not SC) can help for this part: 9

Students' Feedback

Post mid-term exam survey

Question 4 (b) [5 marks]

Let $v_1 = (1, 1, -1)$, $v_2 = (-1, 1, 1)$ and $u_1 = (0, 1, 0)$, $u_2 = (1, 0, -1)$, $u_3 = (1, 1, 1)$.

- (i) Show that $\text{span}\{v_1, v_2\} \subseteq \text{span}\{u_1, u_2, u_3\}$.
- (ii) Is $\text{span}\{v_1, v_2\} = \text{span}\{u_1, u_2, u_3\}$? Justify your answer.

Number of students using GC for this part: 14

Number of students who think GC (but not SC) can help for this part: 14

Students' Feedback

Post mid-term exam survey

GC has improved your performance in this exam.

	No. of students
Agree	12
Disagree	24
Not sure	27
NA	10

Students' Feedback

Post mid-term exam survey

Without GC, you will perform equally well in this exam

	No. of students
Agree	48
Disagree	7
Not sure	18

Students' Feedback

Post mid-term exam survey

Do you think that GC is necessary for this exam?	
	No. of students
Yes	15
No	44
Not sure	14

Students' Feedback

- *It takes a long time to key in matrices in GC*
- *Can't use GC for questions with unknowns*
- *manual calculation can be faster for small matrices*
- *GC will not improve one's performance as workings still need to be shown*
- *GC may discourage us to practice the Gaussian Elimination*
- *SC is equally capable (as GC)*
- *Do not know how to use GC for matrix calculation*
- *No time to use*
- *Save time in performing Gaussian Elimination*
- *GC is a good tool for error checking (23)*

Concluding Remarks

Questions

- Should students be allowed to use GC during exam?
 - **They can be allowed to use GC**
- Will students who use GC during exam have an advantage over students who don't?
 - **Not significant**
- Is it possible to set an exam that is fair to all students with or without using GC?
 - **Yes**

Concluding Remarks

Future improvement of the study

- Post final exam survey
- Comparison between H3 and undergrad
- Comparing the performance of H3 students with and without using GC
- Allowing undergrad to use GC in exam
- Prepare the H3 students on the use of GC