



The Next Wave
-Scientific Calculator for Primary School
Conference

Primary Math, Calculator and I
- some thoughts on teaching and
learning with calculator



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Primary Math and I

- I am teaching university math
- I am not into math education
- My daughter is in K1
- I have been conducting enrichment classes for P5 and P6 students

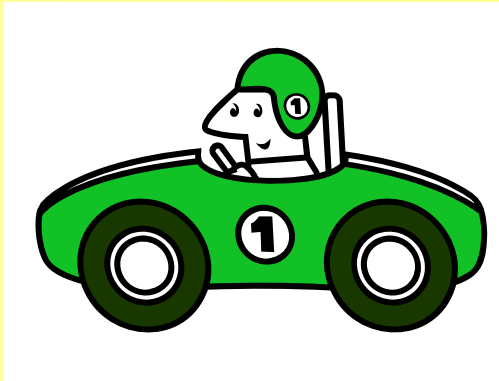


Calculator and I

- I seldom use my calculator
- Use excel file as "calculator"
- Make sure my students bring the allowed calculators to exam
- Decide whether to let students use Graphing calculators
- I use IT in my teaching



Human-beings tend to be lazy



Technologies improve our lives and works if used properly



But over-dependent on technologies may not be a good thing



What will students miss?

- if they are over-dependent on calculator

- to make observation
- to think
- to acquire the skills of solving problems
- to see the beauty of mathematics



My Primary Enrichment Class

- Arithmetic
- Keep away their calculators
- Exercise their brains



Example 1

Tell me the answers in 2 seconds


$$\begin{array}{c} 100 \\ \underbrace{\hspace{10em}} \\ 66 + 47 + 34 + 53 = 200 \\ \underbrace{\hspace{10em}} \\ 100 \end{array}$$

Human brains are more powerful
than calculators

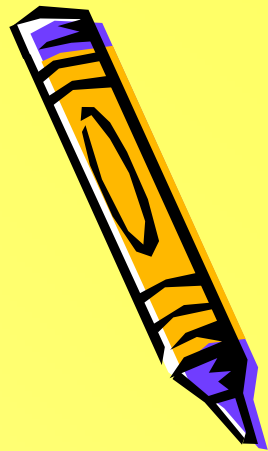


Example 2

$$25 \times 32 \times 125 = 100000$$


$$\underbrace{25 \times 4}_{100} \times \underbrace{8 \times 125}_{1000}$$

Breaking and pairing up



Example 3



$$\begin{array}{r} 199999 + 19999 + 1999 + 199 + 19 \\ + 1 \quad + 1 \quad + 1 \quad + 1 \quad + 1 \\ \hline 200000 + 20000 + 2000 + 200 + 20 \\ \hline 222220 - 5 = 222215 \end{array}$$

Borrow and return



Keep away your calculator

$$66 + 47 + 34 + 53$$

$$25 \times 32 \times 125$$

$$199999 + 19999 + 1999 + 199 + 19$$

Is it really necessary?

For PSLE: **No**

For learning math: **Yes**



Example 3 (revisited)

$$\underbrace{199\dots9}_{100 \text{ digits}} + \underbrace{199\dots9}_{99 \text{ digits}} + \dots + 199 + 19$$

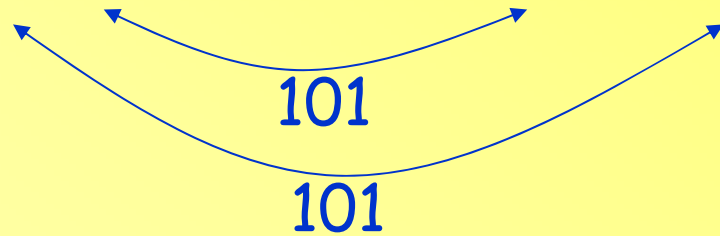
Beyond the capability of the calculator

Borrow and return



Example 4

$$1 + 2 + 3 + \dots + 99 + 100$$



50 pairs of 101: $50 \times 101 = 5050$

key in 100 numbers one at a time





Example 5

Express the quotient in lowest term

$$\frac{\begin{array}{c} \text{transfer 1 over} \\ \boxed{1999} \quad \quad \quad \boxed{1999} \\ 1998 + 1999 + 2000 \end{array}}{\begin{array}{c} \boxed{1999} \quad \boxed{1999} \quad \quad \quad \boxed{1999} \quad \boxed{1999} \\ 1997 + 1998 + 1999 + 2000 + 2001 \\ \text{transfer 1 over} \\ \text{transfer 2 over} \end{array}} = \frac{3 \times 1999}{5 \times 1999}$$



discover the properties of numbers and operations

Trial and Error

- The kids are good at pressing buttons
- Calculator makes it easy to do trial and error
- Minimal thinking



Example 6

What is the digit B ?

$$\begin{array}{r} B 2 \\ \times 7 B \\ \hline 6396 \end{array}$$

Guessing without thinking:
try $B = 1, 2, 3, 4, \dots$

Guessing with thinking:
only try $B = 3$ and 8



Example 7

Find the value of A.

$$\begin{array}{l} 5 \times 5 \quad 7 \times 7 \quad 9 \times 9 \quad A = 5 \times 7 \times 9 = 315 \\ 25 \times 49 \times 81 = A \times A \end{array}$$

$$99225 \xrightarrow{\text{square-root}} 315$$

$$2 \times 2 \times 2 \times 2 \quad 3 \times 3 \times 3 \times 3 \quad 4 \times 4 \times 4 \times 4 \quad B = 2 \times 3 \times 4 = 24$$

$$16 \times 81 \times 256 = B \times B \times B \times B$$

$$331776 \xrightarrow{\text{trial-and-error?}}$$

First Brain, then fingers

- Educate your students to always think first, before using the calculator
- Give your students problems that cannot be solved efficiently without thinking



Example 7 (revisited)

$$\underbrace{16 \times 81 \times 256}_{331776} = B \times B \times B \times B$$

trial-and-error ?

Observe:

1. $20 \times 20 \times 20 \times 20 = 16000$
2. $30 \times 30 \times 30 \times 30 = 81000$

possible values for B: 21, 22, 23, ..., 29





Example 8

Put in the boxes digits 1, 2, 3, 4, 5 such that the product is the greatest.

	4 or 5	2 or 3	1
x		4 or 5	2 or 3

	4	2	1
x		5	3

22313

	4	3	1
x		5	2

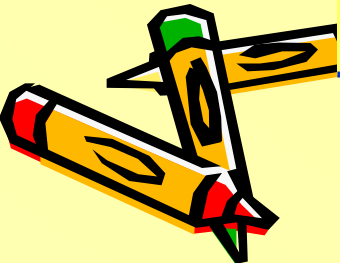
22412

	5	2	1
x		4	3

22403

	5	3	1
x		4	2

22302



Calculator for experiments

- Discover nice patterns in numbers
- Explore large numbers
- Calculators can help with the exploration



Example 9



What is $\underbrace{11\dots1}_{10 \text{ digits}} \underbrace{22\dots2}_{10 \text{ digits}} \div \underbrace{33\dots3}_{10 \text{ digits}} ?$

Ans: $\underbrace{33\dots34}_{9 \text{ digits}}$

$$12 \div 3 = 4$$

$$1122 \div 33 = 34$$

$$111222 \div 333 = 334$$

$$11112222 \div 3333 = 3334$$



Example 10

Last digits of products of 4

$$4 \times 4 = 16$$

odd copies of 4: last digit is 4

$$4 \times 4 \times 4 = 64$$

even copies of 4: last digit is 6

$$4 \times 4 \times 4 \times 4 = 256$$

$$4 \times 4 \times 4 \times 4 \times 4 = 1024$$

$$14 \times 14 = 196$$

$$24 \times 24 = 576$$

$$14 \times 14 \times 14 = 2744$$

$$24 \times 24 \times 24 = 13824$$

:

:



Example 11

Remainders when 3 divides 111...1

$$1 \div 3 \text{ remainder is } 1$$

$$11 \div 3 \text{ remainder is } 2$$

$$111 \div 3 \text{ remainder is } 0$$

$$1111 \div 3 \text{ remainder is } 1$$

$$11111 \div 3 \text{ remainder is } 2$$

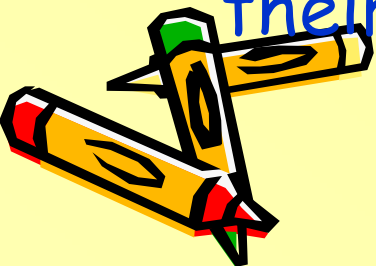
$$111111 \div 3 \text{ remainder is } 0$$



Concluding remarks

When you set a problem:

- don't set it so that the answer can be obtained from the calculator easily
- don't set it so that calculator cannot be used
- set it in such a way that it requires the students to observe and think before using their calculators



Concluding remarks

With the introduction of calculators,

- focus is not on doing computation fast and accurate
- focus is more on thinking and reasoning
- pass the message to your students



Thank you for your attention

