MECHANICAL AND THERMAL ENERGY

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DEFINITIONS OF ENERGY

1 The capacity for work or vigorous activity, strength
2 Exertion of vigor or power
   ‘a project requiring a great deal of time and energy’
3 Usable heat or power
   ‘Each year Americans consume a high percentage of the world’s energy’
4 Physics. The capacity of a physical system to do work -attributive. energy – conservation, efficiency

ENERGY-WORK-TOOL CONCEPT


(old form 5.5-7ky) **Werg** – to do

(suffixed form) **Werg-o**

derivatives handiwork, boulevard, bulwark, **energy**, erg, ergative, -urgy; adrenergic, allergy, argon, cholinergic, demiurge, dramaturge, endergonic, endoergic, energy, ergograph, ergometer, ergonomics, exergonic, exergue, exoergic, georgic, hypergolic, lethargy, liturgy, metallurgy, surgery, synergid, synergism, thaumaturge, **work**

Greek: ergon → energos → energeia → Latin: energia → French: energie
Germanic: werkam → Old High German: werc, Old English: weorc, werc

(zero-grade form) **Wig**

derivatives wrought, irk, wright

(o-grade form) **Worg**

derivatives organ, organon (= **tool**), orgy
**MEASUREMENT**

**Measurement** is the process of assigning numbers to **things** in an **additive way**.

**Counting** assigns nonnegative integers to sets. Is counting a measurement? What does it measure?

Measuring **continuous things** requires **units**. Discuss the process of measuring the mass and volume of things. How do the measured values (assigned values) depend on the units?
A child has 28 blocks – same mass and volume. Her mother discovers that regardless what she does with the blocks, at the end of the day she sees the same number of blocks. Explain how she can use the formula below after some blocks are locked in a box and some blocks are submerged in dirt bathwater.

\[
\text{blocks seen} + \frac{\text{weight of box} - 16 \text{ ounces}}{3 \text{ ounces}} + \frac{\text{height of water} - 6 \text{ inches}}{1/4 \text{ inch}}
\]
PERPETUAL MOTION MACHINES


Energy principles [conservation laws governing the transfer of mechanical & heat energy] can be described in terms of the (ideal) behavior of machines (tools).

A perpetual motion machine is a physical device that can lift an object in one cycle (return to original state).

Jerome Cardan (1501-1576) and Leonardo da Vinci (1452-1519) believed these were impossible; Simon Stevin (1548-1620) and Galileo Galilei (1564-1642) used this to derive statics and dynamics ([3], p28).
WEIGHT-LIFTING MACHINES

We will assume for the remainder of these lectures that *perpetual motion machines are impossible* and follow the logic in [2] I, 4-2.

Consider a **reversible** machine that lifts a three-unit weight a vertical distance $X$ by lowering a one-unit weight a vertical distance 1. Show that no machine can lift a three-unit weight by vertical distance $Y > X$ by lowering a one-unit weight by 1 cm.
WEIGHT LIFTING MACHINES

Show that $3X = 1$. If $3X > 1$  

1. Lift the three unit-weights distance $X$ by lowering the unit-weight by 1 cm
2. Restore the three weights located in the shelves on the right by lowering the weight in the top shelve $3X$ - first lower it by 1 cm to restore the weight on the left to its original position, then lower it by $3X-1$
Consider a set of objects numbered 1, 2, ..., N having weights $W_1, W_2, ..., W_N$ and heights $y_1, y_2, ..., y_N$ and initially at rest. If these objects interact so the total effect only changes their heights, then the weighted sum of heights $\sum_{i=1}^{N} W_i y_i$ remains unchanged.
ELASTIC POTENTIAL ENERGY

Consider the reversible machine that uses a spring to lower a weight by sliding it to the left

Initially, the two weights are placed on each side of the fulcrum so as to balance the lever.
What happens as either weight is moved to the left? Where did the gravitational potential energy go?
WEIGHT DROPPING

Consider dropping an object having weight $W$. What happens to its gravitational potential energy?

Does this violate the conservation of gravitational potential energy?

Discuss the vertical velocity of the object as a function of its weight, initial time, initial height, and time.

\[
\dot{y} = v(W, t_0, y(t_0), t)
\]
GALILEAN INVARIANCE

Show that the function $f$ is invariant under of weight, height, and time translations if and only if the function $v$ has the form

$$v(W, t_0, y(t_0), t) = f(t - t_0)$$

Assume the principle of galilean invariance: physics is the same for people moving with constant velocities. Observe a dropping weight from the ground and from within a moving elevator to show that

$$f(T_1 + T_2) = f(T_1) + f(T_2)$$
ADDITIVE FUNCTIONS

Describe a simple class of functions that satisfies this additive property.

(Hard) Show that every continuous additive function is in this class.

(Brain Bending) Construct an example of an additive function that is not in this class. Hint: R forms a vector space over the field Q of rational numbers and every vector space has a basis.

(Philosophy) What would life be like in such a world?
Convince yourself that $f(s) = -gs$

where $g$ is a constant. What is $g$?

Use the figure below to show that

$$y(t) = y(t_0) - \frac{1}{2}g(t - t_0)^2$$
GRAVITATIONAL MASS AND KINETIC ENERGY

Show that for a dropping weight $y + \frac{1}{2g} \dot{y}^2$ is conserved. Therefore $Wy + \frac{1}{2} M_y \dot{y}^2$ is conserved, where $M_y \equiv W / g$ is, by definition, is the objects gravitational mass.

$\frac{1}{2} M_y \dot{y}^2$ is called the kinetic energy
Consider a spinning pair of objects each of weight $W$.

Define the inertial mass $M_x$ by the equation

$$M_x \ddot{x}^2 = M_y \dot{y}^2$$
SMALL OSCILLATIONS

Consider an object attached to a spring that moves horizontally on the axis. Then for values of $x$ near the equilibrium position $X_0$

$$E \approx \frac{1}{2} M_x \dot{x}^2 + \frac{1}{2} k(x - x_0)^2$$

$$\Rightarrow$$

$$x(t) = x_0 + \sqrt{2E/k} \sin \sqrt{k/M_x} (t - t_0)$$

Period $$T = 2\pi \sqrt{M_x/k}$$
Consider a pendulum - an object on a swinging lever. Then for small $\dot{\theta}$

$$E \approx \frac{L}{2} \left( M_y g \dot{\theta}^2 + M_x L \ddot{\theta}^2 \right)$$

$$\Rightarrow \quad \dot{\theta}(t) = \sqrt{\frac{2E}{LM_y g}} \sin \sqrt{\frac{M_y g}{M_x L}} (t - t_0)$$

Period $T = 2\pi \sqrt{\frac{M_x L}{M_y g}}$
PRINCIPLE OF EQUIVALENCE

There is NO logical reason that inertial mass equals gravitational mass. In other words, it is possible to construct a mathematically consistent model of mechanics in which perpetual motion machines are impossible and galilean invariance holds.

However, Newton, using pendulums, showed they are equal to \(1/1000\) while the Hungarian Baron Eotvos (1889), using a torsion balance, improved this to \(1/3,000,000,000\), and Braginsky and Panov (1971) Improved this by a factor of 30,000.

Einstein showed this has nothing to do with mass!
CONSERVATION OF MOMENTUM

Consider the elastic collision of two objects

Since kinetic energy is conserved

\[ M_1 V_1^2 + M_2 V_2^2 = M_1 V_1'^2 + M_2 V_2'^2 \]

Galilean invariance implies this holds when an arbitrary velocity \( V \) is subtracted from each velocity

Momentum \[ M_1 V_1 + M_2 V_2 = M_1 V_1' + M_2 V_2' \]