
Problem 1. Construct a homeomorphism from the open disk

$$D_1((0,0)) := \{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1 \}$$

onto the open disk with an interval removed

$$D_1((0,0)) \setminus \{(x,0) : x \in [0,1)\}.$$ 

Be sure to prove that the function that you construct satisfies all three conditions required to be a homeomorphism.

Problem 2. Let $\(X, d\)$ be a metric space. A real valued function $f : X \to \mathbb{R}$ is said to be uniformly continuous if there exists a function

$$\delta : (0,\infty) \to (0,\infty)$$

such that for every $\epsilon \in (0,\infty)$

$$d(p, q) < \delta(\epsilon) \text{ implies that } |f(p) - f(q)| < \epsilon, \ p, q \in X.$$ 

Consider $\(0,1\)$ and $\[0,1\]$ to be metric spaces for which $d(p, q) := |p - q|$. Construct a continuous function $f : (0,1) \to \mathbb{R}$ that is not uniformly continuous. Prove that every continuous function $f : [0,1] \to \mathbb{R}$ is uniformly continuous. Explain the difference between these two cases.