Tutorial 6 USC3002 Picturing the World Through Mathematics.
Sem 2/2004

Read pages 40-43 in the textbook and then do the following problems.

**Problem 1.** Consider the plane $\mathbb{R}^2$ and two nonzero vectors $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$. Derive the following formula

$$x_1x_2 + y_1y_2 = ||v_1|| ||v_2|| \cos \theta$$

where $||v_1||$ and $||v_2||$ are the lengths of the vectors and $\theta \in [0, \pi]$ is the angle between them. In your derivation make reference to the Pythagorean Theorem from Euclidean geometry and trigonometry. The common expression is called the dot or scalar or inner product of the two vectors and is commonly denoted by $v_1 \cdot v_2$. If $v_1 \cdot v_2 = 0$ we say that $v_1$ is orthogonal to $v_2$ or that $v_1$ and $v_2$ are orthogonal to each other. A vector that has length one is called a unit vector.

**Problem 2.** A vector $u$ is said to be parallel to a line $L$ or to a line segment $L$ if there exists $p \in L$ such that $p + u \in L$. A vector $v$ is said to be orthogonal to a line $L$ or to a line segment $L$ if $v \cdot u = 0$ whenever $u$ is parallel to $L$. Let $u, v, w$ be unit vectors that are orthogonal to and point inward to the three sides of an equilateral triangle. Prove that if $z$ is a vector that satisfies $z \cdot u \leq 0, z \cdot v \leq 0, z \cdot w \leq 0$ then $z = 0$. Suggestion: choose a specific equilateral triangle with one side along the horizontal axis and the vertex that is opposite this side located at $(0, 1)$. Then compute the coordinates of $u, v, w$. Then show that each of the four vectors having coordinates $(1, 0), (-1, 0), (0, 1), (0, -1)$ can be expressed as a linear combination having nonnegative coefficients of the vectors $u, v, w$. Express $z = (z_1, z_2)$ and then show that $z_1 \leq 0$ and $z_1 \geq 0$ and $z_2 \leq 0$ and $z_2 \geq 0$. What does this imply about the vector $z$?

**Problem 3.** For a triangle $T_1$ construct a sequence of triangulations $T_n$ where $T_{n+1}$ is obtained by connecting the midpoints of each side of each triangle in $T_n$ so that $T_{n+1}$ has four times as many triangles as $T_n$. Prove that the diameters of the triangles in $T_n$ converge uniformly to zero.

**Problem 4.** Do exercises 4, 5, 6, 7 on page 43.