2001/2002 Fall Term MA2108 Advanced Calculus II

Optional Practice Questions for Chapter One

1. For each of the following sequences, either find the limit or show that the limit does not exist.
   (a). $\left\{\dfrac{1}{\sqrt{n+1}(\sqrt{n+8}-\sqrt{n})}\right\}$.
   (b). $\left\{\left(\dfrac{n}{n+3}\right)^{2n}\right\}$.
   (c). $\left\{(n^2+1)\left(1-\cos\dfrac{1}{n}\right)\right\}$.
2. Find the limit of each of the following sequences:
   (a). $a_1 = 1$ and $a_n = \sqrt{1+a_{n-1}}$ $(n = 2, 3, 4, \ldots)$
   (b). $a_1 = \sqrt{2}$ and $a_n = \sqrt{2a_{n-1}}$ $(n = 2, 3, 4, \ldots)$.
   Justify your answers.
3. Let $\{a_n\}$ be a sequence of real numbers. Suppose that both of two subsequences $\{a_{2k-1}\}$ and $\{a_{2k}\}$ converge and $\lim_{k\to\infty} a_{2k-1} = \lim_{k\to\infty} a_{2k} = A$. Show that $\lim_{n\to\infty} a_n = A$.
4. Let $\{a_n\}$ be a bounded sequence of nonnegative numbers. Prove that
$$\overline{\lim_{n\to\infty}} a_n^2 = \left(\overline{\lim_{n\to\infty}} a_n\right)^2.$$
5. Prove that a sequence $\{a_n\}$ converges to 0 if and only if the sequence of absolute values $\{|a_n|\}$ converges to 0.
6. Let A and B be two non-empty bounded set of real numbers
   (a). Show that $\inf A \cup B = \min\{\inf A, \inf B\}$.
   (b). Is it true that $\inf A \cap B = \max\{\inf A, \inf B\}$? Justify your answer.

Some suggested answers: