

1999/2000 Special Term **MA2108 Advanced Calculus II** Midterm Exam

29 May 2000-Time allowed: 2 hours

Tutorial Group:

Thursday 11-1

Friday 2-4

Friday 4-6

Name:

ID number:

Question 1[30 Marks, 5 marks for each]

For each of the following sequences, either find the limit or show that the limit does not exist

$$(1). \sin \frac{2n\pi+1}{6n+5} + \cos \frac{n^2\pi+1}{2n^2+3n+2}$$

$$(2). \left(1 - \frac{2}{n+1}\right)^{3n}$$

$$(3). \left\{ \frac{1}{\sqrt{n}(\sqrt{n+1}-\sqrt{n})} \right\}$$

$$(4). \left\{ \left(\frac{2n+1}{3n+2} \right)^n \right\}$$

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$$(5). \left\{ (n^2 + 1) \sin \frac{1}{n^2} \right\}$$

$$(6). \left\{ \frac{(n+2)! + n}{n(2^n + n!)} \right\}$$

Question 2[30 Marks, 5 marks for each] Determine the convergence or divergence of each of the following series. Justify your answers.

(1). $\sum_{n=2}^{\infty} \frac{1}{n-1}$.

(2). $\sum_{n=1}^{\infty} \frac{1}{2n^2+1}$.

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$$(3). \sum_{n=1}^{\infty} \frac{n}{2^n}.$$

$$(4). \sum_{n=1}^{\infty} \sqrt{\frac{2^n + n^2}{3^n + n}}.$$

$$(5). \sum_{n=1}^{\infty} \frac{1}{n(\ln^2 n + 1)}.$$

$$(6). \sum_{n=3}^{\infty} \frac{\sin n}{n \ln n (\ln(\ln n))^2}.$$

Question 3.[10 marks]

Consider the sequence $\{a_n\}$ given by

$$a_1 = \sqrt{2}, \quad a_n = \sqrt{2a_{n-1}} \quad n = 2, 3, 4, \dots .$$

Show that $\{a_n\}$ converges, and find the limit.

Question 4.[15 marks]

Let A and B be two non-empty bounded sets of real numbers.

- (1). Show that $\inf A \cup B = \min\{\inf A, \inf B\}$. [10 marks]
- (2). Is it true $\inf A \cap B = \max\{\inf A, \inf B\}$? Justify your answer. [5 marks]

Question 5.[15 marks]

- (1). State, without proof, Cauchy's criterion for sequence. [5 marks]
- (2). Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences. Show that $\{a_n + b_n\}$ is convergent by using the Cauchy criterion. [5 marks]
- (3). Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences. Show that $\{a_n b_n\}$ is convergent by using the Cauchy criterion. [5 marks]