1. Prove the following limits by using $\epsilon - N$ definition
   i) $\lim_{n\to\infty} \frac{2n^2}{3n^2+2} = \frac{2}{3}$.
   ii) $\lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = 1$.

2. For each of the following statements, either prove that the statement is true or give a counter example to show that the statement is false:
   i) If $\{a_n\}$ converges and $\{b_n\}$ diverges, then $\{a_n - b_n\}$ diverges.
   [Hint: You may use Theorem 1.4.3.]
   ii) If $\{a_n\}$ converges and $\{b_n\}$ diverges, then $\{a_n b_n\}$ diverges.

[From Question 3 onwards, you may assume the limits of the standard sequences.]

3. Evaluate the following limits:
   i) $\lim_{n\to\infty} \frac{n^2+5n^3-1}{3n^3+6n+4}$;
   ii) $\lim_{n\to\infty} \frac{3^n+n^8}{2n^2+7^n}$;
   iii) $\lim_{n\to\infty} \sqrt{\frac{n^4+4n^3+1}{n^3+2n^2}}$.

4. Use the Squeeze theorem to find the following limits:
   i) $\lim_{n\to\infty} \frac{1+|\sin n|}{2n}$;
   ii) $\lim_{n\to\infty} \left(\frac{2n-5}{3n+1}\right)^n$.

5. Do the following sequences tend to $+\infty$ or $-\infty$? Justify your answer.
   i) $\left\{\frac{e^n}{n^{100}}\right\}$;
   ii) $\left\{\frac{n}{\ln\frac{1}{n+2}}\right\}$.

6. Evaluate the following limits (you may assume the limits of the standard sequences and use the Squeeze theorem, etc.)
   (a). $\lim_{n\to\infty} \sqrt[3]{\frac{2n^4+n+1}{16n^4+n^2+2}}$;
   (b). $\lim_{n\to\infty} \left(3 + \ln(\cos\frac{1}{\sqrt{n}}) + \frac{n^2}{1.1^n}\right)$;
   (c). $\lim_{n\to\infty} \frac{n^4+8^n}{9^n+n+8^n}$;
   (d). $\lim_{n\to\infty} \sqrt[n]{\frac{n}{3}}$;
   (e). $\lim_{n\to\infty}(\sqrt{3n-2} - \sqrt{3n-3})$;
   (f). $\lim_{n\to\infty} \left(\frac{3+(-1)^n}{5}\right)^n$;

(g). $\lim_{n\to\infty} \frac{7^n+\ln n-n!}{n!+n^2}$;
(h). $\lim_{n\to\infty} \frac{n^{100}100^n}{n!}$;
(i). $\lim_{n\to\infty} \frac{(-1)^{n+1}\ln n}{\sqrt{n}}$;
(j). $\lim_{n\to\infty}(3^n+4^n)^{\frac{1}{n}}$;
(k). $\lim_{n\to\infty} n\sin\frac{3}{n}$; (Hint: $\lim_{x\to 0}\frac{\sin x}{x}=1$.)
(l). $\lim_{n\to\infty}\left(1-\frac{1}{3n}\right)^{2n}$;
(m). $\lim_{n\to\infty}\left(1+\frac{1}{n^3}\right)^{n^3+2}$.

7. Let S and T be two bounded sets of real numbers. Show that $S\cup T$ is also a bounded set.

8. i) Show that a sequence $\{a_n\}$ is bounded if and only if $\{|a_n|\}$ is bounded.
   ii) Using i) or otherwise, show that if $\lim_{n\to\infty} a_n=0$ and $\{b_n\}$ is bounded, then $\lim_{n\to\infty} a_nb_n=0$.

Some suggested answers:
2. i) True ii) False counter-example: $a_n=1/n$, $b_n=n$.
3. i) $\frac{5}{3}$ ii) 0 iii) 0.
4. i) 0 ii) 0
5. i) $\lim_{n\to\infty}\frac{e^n}{n^{100}}=+\infty$ ii) $\lim_{n\to\infty}\frac{n}{\ln\frac{1}{n+2}}=-\infty$.
6. (a) 1/2 (b) 3 (c) 0 (d) 1 (e) 0 (f) 0 (g) -1 (h) 0 (i) 0 (j) 4 (k) 3 (l) $e^{-\frac{2}{3}}$ (m) e.