1. i) By integrating from $t = 0$ to $t = x$ the power series $\dfrac{1}{1+t} = \sum_{n=0}^{\infty}(-1)^n t^n$, $|t| < 1$, show that

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$$

for all $|x| < 1$.

ii) Use part i) to find the Taylor series of $\ln(1+2x^2)$ at $x_0 = 0$.

2. Use series to estimate the integrals' values with an error of magnitude less than 10^{-8}.

i) $\displaystyle\int_0^{0.2} \sin x^2 dx$.

ii) $\displaystyle\int_0^{0.1} \frac{1}{\sqrt{(1+x^4)}} dx$.

3. Use series to evaluate the limits

i) $\dfrac{\arctan y - \sin y}{y^3 \cos y}$.

ii) $\displaystyle\lim_{x\to\infty} x^2(e^{-1/x^2} - 1)$.

4.

i) According to the Alternating Series Estimation Theorem, how many terms of the Maclaurin series for $\arctan 1$ would you have to add to be sure of finding $\dfrac{\pi}{4}$ with an error of magnitude less than 10^{-3}? Give reasons for your answer.

ii) Find the Maclaurin series of $\arcsin x$, and estimate $\dfrac{\pi}{6}$ with an error of magnitude less than 10^{-3}.

5. Let $f(x) = \sqrt[5]{1+x^3}$. Find $f^{(30)}(0)$.

6. Estimate $\sqrt{101}$ with an error of magnitude less than 10^{-6}.