1. For each of the following series, calculate the n-th partial sum S_n, and determine whether the series is convergent or divergent

   i) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$.

   ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

2. Determine the convergence or divergence of each of the following series. Justify your answers.

   (a). $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{2n^2+n}$.

   (b). $\displaystyle\sum_{n=1}^{\infty} \sin \frac{n\pi}{2}$.

   (c). $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1+\ln n}{n+n^3+4}$.

   (d). $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{n^2}$.

   (e). $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3}{3^{n+1}-n}$.

   (f). $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^{1+\frac{1}{n}}}$.

   (g). $\displaystyle\sum_{n=1}^{\infty} \frac{4+(-1)^n}{2n}$.

   (h). $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(1+\ln n)^p}$ with $p \le 0$.

   (i). $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

Some suggested answers:

1. i) $S_n = \ln 3 - \ln(n+3)$ and $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$ is divergent.

1. ii) $S_n = \dfrac{1}{2}\left[\dfrac{3}{2} - \dfrac{2n+3}{(n+1)(n+2)}\right]$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \lim S_n = \frac{3}{4}$.

2. a) divergent by the divergence test.

2. b) divergent by the divergence test.

2. c) divergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).

2. d) convergent by the comparison test (comparing with $\sum_{n=1}^{\infty} \frac{4}{n^2}$).

2. e) convergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} (\frac{2}{3})^n$).

2. f) divergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).

2. g) divergent by the comparison test (comparing with $\sum_{n=1}^{\infty} \frac{3}{2n}$).

2. h) divergent by the comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).

2. i) divergent by the limit comparison test (comparing with $\sum_{n=1}^{n} \frac{1}{n}$).