1. Evaluate the limits. Justify your answers.

i) $\lim_{n\to\infty} \int_0^1 \frac{n+e^x}{n+x^2} dx.$

ii) $\lim_{n\to\infty} \int_1^2 \left(\frac{x^2+1}{8}\right)^n \sin nx dx.$

2. Prove that each of the following series of functions converges uniformly on the indicated interval.

i) $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2+x^2}$, $x \in (-\infty, +\infty)$.

ii) $\sum_{n=1}^{\infty} \frac{1}{1+n^3x^2}$, $x \in [2, \infty)$.

iii) $\sum_{n=1}^{\infty} \frac{xe^{-nx}}{n^2}$, $x \in (0, \infty)$.

3. Consider the function

$$F(x) = \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{1+x^{2k}}, \quad x \in \left(0, \frac{2}{3}\right).$$

Show that F is continuous on the interval $\left(0, \frac{2}{3}\right)$.