1. Evaluate $\displaystyle\sum_{n=0}^{\infty}\int_0^{\frac{1}{2}}\frac{x^n(1-x^2)}{\sqrt{1+x}}dx$ in simplest form. Justify your answer.

2. Let $\displaystyle\sum_{k=1}^{\infty}a_k$ be an absolutely convergent series.

i) Show that $\displaystyle\sum_{k=1}^{\infty}a_k\sin kx$ converges uniformly on $(-\infty,+\infty)$.

ii) Hence evaluate $\displaystyle\int_0^{2\pi}\sum_{k=1}^{\infty}a_k\sin kx dx$. Justify your answer.

3. Estimate the integral $\displaystyle\int_0^1 e^{-x^3}dx$ such that the error is within 0.001.

(Hint: $e^x=\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$.)

4. Show that the function $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{\cos^n x}{n^3}$ is differentiable on $(-\infty,+\infty)$.

5. Show that the function $f(x)=\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$ is differentiable on $(-\infty,+\infty)$. Furthermore show that $y=f(x)$ is a solution to the differential equation $y''=y$.

6. Show that the series of functions $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$ DOES NOT converge uniformly on $(-\infty,+\infty)$.

(Hint: Let $S(x)=\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$ and $S_n(x)=\displaystyle\sum_{k=0}^{n}\frac{x^k}{k!}$. Then $|S_n(x)-S(x)|\geq\dfrac{x^{n+1}}{(n+1)!}$ for $x>0$.)