

1. Find the radius of convergence of each of the following power series:

i) $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^{n^2} x^n.$

ii) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!} (x-1)^n.$

iii) $\frac{x}{5} + \left(\frac{x}{6}\right)^2 + \left(\frac{x}{5}\right)^3 + \left(\frac{x}{6}\right)^4 + \left(\frac{x}{5}\right)^5 + \left(\frac{x}{6}\right)^6 + \dots.$

iv) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2}.$

2. Find the interval of convergence of each of the following power series:

i) $\sum_{n=1}^{\infty} \frac{(-2x)^n}{n^{\frac{3}{2}}}.$

ii) $\sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n+1}.$

iii) $\sum_{n=1}^{\infty} \frac{(1-3x)^n}{n}.$

3. By computing derivatives, find the Taylor series of

i) $f(x) = e^{2x}$ at $x = 3.$

ii) $f(x) = \cos x$ at $x = \frac{\pi}{3}.$