# Take-home Exam 1

**Question 1.** [20 marks] Prove the following limits by using $\epsilon - N$ definition

i) $\lim\limits_{n\to\infty} \dfrac{3n+8}{2n+9} = \dfrac{3}{2}$.

ii) $\lim\limits_{n\to\infty} \dfrac{(-1)^n n}{n^2+1} = 0$.

**Question 2.** [40 marks] For each of the following sequences, either find the limit or show that the limit does not exist.

(a) $\left\{\left(\sqrt{n^2+n}-n\right)\right\}$.

(b) $\left\{(2^n+3^n)^{\frac{1}{n}}\right\}$.

(c) $\left\{\sqrt[4]{\dfrac{n!+2n^5+\ln n}{n!+5^n+3n}}\right\}$.

(d) $\left\{\left(\dfrac{3n}{3n-1}\right)^{2n+\sqrt{n}}\right\}$.

(e) $\left\{\dfrac{n^{50}\cdot 50^n\cdot \sin n}{n!}\right\}$.

**Question 3.** [20 marks] Let $f$ and $g$ be real-valued function defined on a nonempty set $E$. Prove each of the following.

(a) If $f(x) \le g(x)$ for all $x \in E$, then $\inf\{f(x) \mid x \in E\} \le \inf\{g(x) \mid x \in E\}$.

(b) $\inf\{f(x) \mid x \in E\} + \inf\{g(x) \mid x \in E\} \le \inf\{f(x)+g(x) \mid x \in E\}$.

**Question 4.** [20 marks] For $A$ and $B$, subsets of $\mathbb{R}$, define $A+B = \{a+b \mid a \in A,\ b \in B\}$. Prove that

$$\sup(A+B) = \sup A + \sup B.$$