1. Denote the set of rational numbers by $\mathbb{Q}$. Consider the set
$$S = \{x \in \mathbb{Q} \mid 0 \leq x < 1\}.$$
Find $\sup S$ and $\inf S$. Justify your answers.
2. Let A and B be two non-empty bounded set of real numbers such that $A \subseteq B$. Show that $\inf A \geq \inf B$.
3. Let A and B be two non-empty bounded set of real numbers
   i) Show that $\sup A \cup B = \max\{\sup A, \sup B\}$.
   ii) Is it true that $\sup A \cap B = \min\{\sup A, \sup B\}$? Justify your answer.
4. Consider the sequence $\{a_n\}$ defined recursively by
$$a_1 = 2, \quad a_n = \sqrt{6 + a_{n-1}}, \quad n = 2, 3, 4, \cdots .$$
   i) Show that $2 \leq a_n \leq 3$ for all n.
   ii) Show that $\{a_n\}$ is monotone increasing.
   iii) Using parts i) and ii), show that $\{a_n\}$ converges, and find its limit.
5. Consider the sequence $\{x_n\}$ defined recursively by
$$x_1 = \frac{3}{4}, \quad x_{n+1} = 2x_n - x_n^2, \quad n = 1, 2, 3, \cdots .$$
Show that $\{x_n\}$ converges, and find its limit. (Hint: Show that $x_n \leq 1$ for all n and $\{x_n\}$ is monotone increasing.)
6. Find the $\overline{\lim}$ and $\underline{\lim}$ of the sequences:
   (a). $\left\{4 + \cos\frac{n\pi}{2}\right\}$.
   (b). $\left\{\frac{1 + (-1)^n}{n}\right\}$.

Some suggested answers:

1. $\sup S = 1$ and $\inf S = 0$.
4. $\lim_{n\to\infty} a_n = 3$.
5. $\lim_{n\to\infty} x_n = 1$.
6. a) $\overline{\lim} = 5$ and $\underline{\lim} = 3$.
6. b) $\overline{\lim} = \underline{\lim} = \lim = 0$.