

1. Find the limit inferior and limit superior of the following sequences

a) $\left\{ \frac{2 - (-1)^n n}{4n + 2} \right\}$.

b) $\left\{ \left(0.9 + \sin \frac{n\pi}{2} \right)^n \right\}$.

c) $\sqrt[n]{\frac{(n!)^2}{(2n)!}}$. (Hint: Use Exercise 8.2 in the lecture notes.)

2. Let $\{a_n\}$ be a bounded sequence of real numbers. Show that

$$\limsup_{n \rightarrow \infty} \sqrt{|a_n|} = \sqrt{\limsup_{n \rightarrow \infty} |a_n|}.$$

3. Let $\{a_n\}$ and $\{b_n\}$ be Cauchy sequences. Show that $\{a_n + b_n\}$ and $\{a_n b_n\}$ are also Cauchy sequences.

4. For each of the following series, calculate the n -th partial sum S_n , and determine whether the series is convergent or divergent.

i) $\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$.

ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.