1. Determine the convergence or divergence of each of the following series. Justify your answers.

   (a). $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{2n^2+n}$.

   (b). $\displaystyle\sum_{n=1}^{\infty} \sin\frac{n\pi}{2}$.

   (c). $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1+\ln n}{n+n^3+4}$.

   (d). $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{n^2}$.

   (e). $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3}{3^{n+1}-n}$.

   (f). $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^{1+\frac{1}{n}}}$.

   (g). $\displaystyle\sum_{n=1}^{\infty} \frac{4+(-1)^n}{2n}$.

   (h). $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(1+\ln n)^p}$ with $p \leq 0$.

   (i). $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

2. (a). Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a convergent series with each $a_n \geq 0$. Show that the series $\displaystyle\sum_{n=1}^{\infty} a_n^2$ is also convergent.
   [Hint: First show that $|a_n| < 1$ for n sufficiently large, and then use the comparison test.]

   (b). Give an example of a convergent series $\displaystyle\sum_{n=1}^{\infty} a_n$ such that $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges.

3. Use the integral test to determine the convergence or divergence of the series:

   (a). $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$.

   (b). $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n[1+(\ln n)^2]}$.

Some suggested answers:

1. a) divergent by the divergence test.

1. b) divergent by the divergence test.

1. c) divergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).

1. d) convergent by the comparison test (comparing with $\sum_{n=1}^{\infty} \frac{4}{n^2}$).

1. e) convergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$).

1. f) divergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).

1. g) divergent by the comparison test (comparing with $\sum_{n=1}^{\infty} \frac{3}{2n}$).

1. h) divergent by the comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).

1. i) divergent by the limit comparison test (comparing with $\sum_{n=1}^{\infty} \frac{1}{n}$).