

1. Use the ratio test to determine the convergence or divergence of each of the following series.

(a). 
$$\sum_{n=1}^{\infty} \frac{(3n)!}{6^n n! (2n)!}$$

(b). 
$$\sum_{n=1}^{\infty} a_n, \text{ where } a_1 = 1, a_n = 2 \left(1 - \frac{1}{n}\right)^n a_{n-1}, n = 2, 3, \dots$$

2. Use the (simplified) root test to determine the convergence or divergence of each of the following series.

(a). 
$$\sum_{n=1}^{\infty} \frac{5n^2 \cdot 3^n}{4^{n+4}}$$

(b). 
$$\sum_{n=1}^{\infty} \frac{3^{2n}}{5^n} \left(1 - \frac{1}{2n}\right)^{n^2}$$

(c). 
$$\frac{1}{4} + \frac{1}{5^2} + \frac{1}{4^3} + \frac{1}{5^4} + \frac{1}{4^5} + \frac{1}{5^6} + \frac{1}{4^7} + \frac{1}{5^8} + \dots$$

3. Determine the convergence or divergence of each of the following series. Justify your answers.

(a). 
$$\sum_{n=1}^{\infty} (\sqrt{2n+2} - \sqrt{n})$$

(b). 
$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} \cdot \frac{2^n}{5^n}$$

(c). 
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.2}}$$

(d). 
$$\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{n^2}$$

(e). 
$$\sum_{n=2}^{\infty} \frac{1}{(\ln n)^3}$$

(f). 
$$\sum_{n=1}^{\infty} \left(\frac{4}{9} + \frac{n^3}{3^n}\right)^{\frac{n}{2}}$$

4. Consider the series 
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{\sqrt{n}}$$

- i) Use the alternating series test to show that the series is convergent.
- ii) Using part i) or otherwise, show that the series is conditionally convergent.