1. Show that the series $\sum_{n=1}^{\infty}(-1)^n \frac{\cos n}{2^n}$ is absolutely convergent.
2. For each of the following series, determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your answers.
   (a). $\sum_{n=1}^{\infty}(-1)^n \frac{3}{2n+1}$.
   (b). $\sum_{n=1}^{\infty}(-1)^n \frac{n}{4n+3}$.
   (c). $\sum_{n=1}^{\infty}(-1)^n \left(\frac{1+2n}{3+4n}\right)^n$.
   (d). $\sum_{n=2}^{\infty}(-1)^{n+1} \frac{\cos n}{n(\ln n)^2}$.
3. Estimate the infinite sum $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n^5}$ such that the error is within 0.001.
4. For each of the following sequence of functions, determine whether is converges pointwise to a function, and find the limiting function if it exists. Justify your answers.
   (a). $\left\{\left(1+\frac{x}{n}\right)^{nx}\right\}$, $x \in (-\infty, +\infty)$.
   (b). $\{x^{n+1}\}$, $x \in [-1, 1]$.
   (c). $\left\{\frac{x^{2n}}{1+x^{2n}}\right\}$, $x \in [0, 1]$.
5. Let $\{F_n\}$ be a sequence of functions on an interval I. It is given that $\{F_n\}$ converges uniformly on some function F on I. Suppose also that for each $n \in \mathbb{Z}^+$, there exists a real number $M_n > 0$ such that
$$|F_n(x)| \leq M_n \quad \text{for all } x \in I.$$
   Show that there exists a real number M such that $|F(x)| \leq M$ for all $x \in I$.