

1. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.

(a).  $F_n(x) = \frac{n^2}{n^2 + x^2}, x \in [0, 1]$ .

(b).  $F_n(x) = x^n(1 - x), x \in [0, 1]$ .

(c).  $f_n(x) = \frac{n \ln x}{x^n}, x \in [1, \infty)$ .

(d).  $f_n(x) = \frac{n \ln x \cos nx}{x^n}, x \in [4, \infty)$ .

(e).  $F_n(x) = \frac{n^2}{n^2 + x^2}, x \in [0, +\infty)$ .

2. Prove that each of the following series of functions converges uniformly on the indicated interval.

i)  $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2 + x^2}, x \in (-\infty, +\infty)$ .

ii)  $\sum_{n=1}^{\infty} \frac{1}{1 + n^3 x^2}, x \in [2, \infty)$ .

iii)  $\sum_{n=1}^{\infty} \frac{x e^{-nx}}{n^2}, x \in (0, \infty)$ .

3. Let  $\sum_{n=1}^{\infty} f_n(x)$  and  $\sum_{n=1}^{\infty} g_n(x)$  be series of functions on an interval  $I$  with

$$|f_n(x)| \leq g_n(x)$$

for all  $x \in I$  and  $n \geq 1$ . Suppose that the series of functions  $\sum_{n=1}^{\infty} g_n(x)$

converges uniformly. Show that the series of functions  $\sum_{n=1}^{\infty} f_n(x)$  converges uniformly.

4. Does the series of functions  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^x}$  converge uniformly on the interval  $(0, +\infty)$ ? Justify your answer.