1. Evaluate the limits. Justify your answers.

   i) $\lim_{n\to\infty} \int_0^1 \frac{n+e^x}{n+x^2}dx.$

   ii) $\lim_{n\to\infty} \int_1^2 \left(\frac{x^2+1}{8}\right)^n \sin nxdx.$

2. Consider the function
$$F(x) = \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{1+x^{2k}}, \quad x \in \left(0, \frac{2}{3}\right).$$
   Show that $F$ is continuous on the interval $\left(0, \frac{2}{3}\right)$.

3. Evaluate $\sum_{n=0}^{\infty} \int_0^{\frac{1}{2}} \frac{x^n(1-x^2)}{\sqrt{1+x}}dx$ in simplest form. Justify your answer.

4. Let $\sum_{k=1}^{\infty} a_k$ be an absolutely convergent series.

   i) Show that $\sum_{k=1}^{\infty} a_k \sin kx$ converges uniformly on $(-\infty, +\infty)$.

   ii) Hence evaluate $\int_0^{2\pi} \sum_{k=1}^{\infty} a_k \sin kxdx$. Justify your answer.

5. By using the formulate $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, estimate the integral $\int_0^1 e^{-x^3}dx$ such that the error is within 0.001.