Take-home Exam 3

In order to get the full marks for the project of take-home exams, you need to have 50 or more points. Well there are 10 points in this take-home exam.

Question 1. [2 points, 1 for each part]

(a) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges, prove that $\sum_{n=1}^{\infty}(a_n + b_n)$ diverges.

(b) If a and b are positive real numbers, prove that

$$\sum_{k=1}^{\infty} \frac{1}{(ak+b)^p}$$

converges if $p > 1$ and diverges if $p \leq 1$.

Question 2. [3 points, 1 for each part] Test the series for convergence or divergence.

(a) $\sum_{k=1}^{\infty}(-1)^k 2^{1/k}$.

(b) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{(k+1)(k+2)}$.

(c) $\sum_{k=1}^{\infty}(\sqrt[k]{2} - 1)$.

(Hint: Try the limit comparison test with the harmonic series. Use $\lim_{k\to\infty} \frac{2^{1/k} - 1}{1/k} = \lim_{x\to 0} \frac{2^x - 1}{x}$ and then use L'Hospital rule for finding the limit.)

Question 3 [5 points, 1 for each part]

Determine the absolute convergence, conditional convergence or divergence of each of the following series. Justify your answers.

(a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}+1}$.

(b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{2k^2 + (-1)^k}$.

(c) $\sum_{k=1}^{\infty} \frac{\sin kt}{k^2+3}$, $\quad t \in \mathbb{R}$.

(d) $\sum_{k=2}^{\infty} \frac{(-1)^k \ln(\ln k)}{\sqrt{\ln k} + 1}$.

(e) $\sum_{k=1}^{\infty} \frac{(-1)^k k^k}{(k+1)^k}$.