2003/2004 Semester I MA2108 Advanced Calculus II

Take-home Exam 6

In order to get the full marks for the project of take-home exams, you need to have 50 or more points. Well there are 10 points in this take-home exam.

Question 1. [4 points, 1 for each part] For each of the following, determine all values of x for which the given series converges.

(a) $\displaystyle\sum_{k=1}^{\infty} \frac{(1-2x)^k}{k}$.

(b) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k}{k^3}(2x+1)^k$.

(c) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k x^k}{2^k(1-x)^k}, \quad x \neq 1$.

(Hint: Let $t = \dfrac{3x}{2(1-x)}$.)

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{kx^k}, \quad x \neq 0$.

(Hint: Let $t = \dfrac{1}{x}$.)

Question 2. [4 points, 1 for each part] Using any applicable method, find the Taylor series of each of the following functions at the indicated point, and specify the interval on which the series converges to the function.

(a) $f(x) = \cos x^2, \quad x_0 = 0$.

(b) $f(x) = \ln\left(\dfrac{1+x}{1-x}\right), \quad x_0 = 0$.

(c) $f(x) = \sqrt{x}, \quad x_0 = 1$.

(d) $f(x) = \dfrac{x^2}{1-x^2}, \quad x_0 = 0$.

Question 3. [2 points, 1 for each part]

(a) Use series to estimate the integral's value

$$\int_0^{0.1} \arctan x^2 \, dx$$

with an error of magnitude less than 10^{-8}.

(b) Let $f(x) = x^3 \sin x^9$. Find $f^{(30)}(0)$.