1. Find the limit inferior and limit superior of the following sequences

   a) $\left\{ \dfrac{2-(-1)^n n}{4n+2} \right\}$.

   b) $\left\{ \left(0.9 + \sin \dfrac{n\pi}{2}\right)^n \right\}$.

   c) $\sqrt[n]{\dfrac{(n!)^2}{(2n)!}}$. (Hint: Use Exercise 8.2 in the lecture notes.)

2. Let $\{a_n\}$ be a bounded sequence of real numbers. Show that

$$\limsup_{n\to\infty} \sqrt{|a_n|} = \sqrt{\limsup_{n\to\infty} |a_n|}.$$

   (Hint: Let $b_n = \sup\{|a_n|, |a_{n+1}|, |a_{n+2}|, \ldots\}$ and let

$$B_n = \sup\{\sqrt{|a_n|}, \sqrt{|a_{n+1}|}, \sqrt{|a_{n+2}|}, \ldots\}.$$

   Recall from the definition that $\overline{\lim_{n\to\infty}} \sqrt{|a_n|} = \lim_{n\to\infty} B_n$ and $\overline{\lim_{n\to\infty}} |a_n| = \lim_{n\to\infty} b_n$. Prove that $B_n \le \sqrt{b_n}$ and $\sqrt{b_n} \le B_n$.)

3. Let $\{a_n\}$ and $\{b_n\}$ be Cauchy sequences. Show that $\{a_n + b_n\}$ and $\{a_n b_n\}$ are also Cauchy sequences.

4. For each of the following series, calculate the n-th partial sum S_n, and determine whether the series is convergent or divergent.

   i) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$.

   ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.