

1. Determine the convergence or divergence of each of the following series. Justify your answers.

(a). $\sum_{n=1}^{\infty} \frac{n^2 - 1}{2n^2 + n}$.

(b). $\sum_{n=1}^{\infty} \sin \frac{n\pi}{2}$.

(c). $\sum_{n=1}^{\infty} \frac{n^2 + 1 + \ln n}{n + n^3 + 4}$.

(d). $\sum_{n=1}^{\infty} \frac{3 + \sin n}{n^2}$.

(e). $\sum_{n=1}^{\infty} \frac{2^n + 3}{3^{n+1} - n}$.

(f). $\sum_{n=1}^{\infty} \frac{2}{n^{1+\frac{1}{n}}}$.

(g). $\sum_{n=1}^{\infty} \frac{4 + (-1)^n}{2n}$.

(h). $\sum_{n=2}^{\infty} \frac{1}{n(1 + \ln n)^p}$ with $p \leq 0$.

(i). $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$.

2. (a). Let $\sum_{n=1}^{\infty} a_n$ be a convergent series with each $a_n \geq 0$. Show that the series

$$\sum_{n=1}^{\infty} a_n^2$$

is also convergent.

[Hint: First show that $|a_n| < 1$ for n sufficiently large, and then use the comparison test. Another solution is to use limit comparison test.]

- (b). Give an example of a convergent series $\sum_{n=1}^{\infty} a_n$ such that $\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges.

3. Use the integral test to determine the convergence or divergence of the series:

(a). $\sum_{n=1}^{\infty} \frac{1}{n(1 + \ln n)}$.

(b). $\sum_{n=1}^{\infty} \frac{1}{n[1 + (\ln n)^2]}$.