

1. Evaluate the limits. Justify your answers.

i)  $\lim_{n \rightarrow \infty} \int_0^1 \frac{n + e^x}{n + x^2} dx.$

ii)  $\lim_{n \rightarrow \infty} \int_1^2 \left( \frac{x^2 + 1}{8} \right)^n \sin nx dx.$

2. Consider the function

$$F(x) = \sum_{k=1}^{\infty} \frac{(-1)^k x^k}{1 + x^{2k}}, \quad x \in \left( 0, \frac{2}{3} \right).$$

Show that  $F$  is continuous on the interval  $\left( 0, \frac{2}{3} \right).$

3. Evaluate  $\sum_{n=0}^{\infty} \int_0^{\frac{1}{2}} \frac{x^n (1 - x^2)}{\sqrt{1 + x}} dx$  in simplest form. Justify your answer.

4. Let  $\sum_{k=1}^{\infty} a_k$  be an absolutely convergent series.

i) Show that  $\sum_{k=1}^{\infty} a_k \sin kx$  converges uniformly on  $(-\infty, +\infty).$

ii) Hence evaluate  $\int_0^{2\pi} \sum_{k=1}^{\infty} a_k \sin kx dx.$  Justify your answer.

5. By using the formulae  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ , estimate the integral  $\int_0^1 e^{-x^3} dx$  such that the error is within 0.001.