

1. Show that the function  $f(x) = \sum_{n=1}^{\infty} \frac{\cos^n x}{n^3}$  is differentiable on  $(-\infty, +\infty)$ .
2. (a). Prove that the function

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad (|x| < \infty)$$

is a solution of the differential equation

$$\frac{dy}{dx} = y.$$

- (b). By solving the above differential equation, show that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad (|x| < \infty).$$

3. Find the radius of convergence of each of the following power series:

- i)  $\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^{n^2} x^n.$

- ii)  $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!} (x-1)^n.$

- iii)  $\frac{x}{5} + \left(\frac{x}{6}\right)^2 + \left(\frac{x}{5}\right)^3 + \left(\frac{x}{6}\right)^4 + \left(\frac{x}{5}\right)^5 + \left(\frac{x}{6}\right)^6 + \cdots.$

- iv)  $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2}.$

4. Estimate  $\sqrt[3]{9}$  with an error of magnitude less than  $10^{-4}$ . (Hint:  $\sqrt[3]{9} =$

$$(8+1)^{\frac{1}{3}} = 2 \left(1 + \frac{1}{8}\right)^{\frac{1}{3}}.)$$