

MA2108

Midterm

5 October 2004

Time allowed: 1.5 hours

Tutorial Group:(circle one)

Tuesday 3-4 (ST1)

Tuesday 4-5 (ST2)

Wednesday 11-12 (ST3)

Wednesday 12-1 (ST4)

Thursday 11-12 (ST5)

Thursday 12-1 (ST6)

Friday 12-1 (ST7)

Friday 1-2 (ST8)

Saturday 11-12 (ST9)

Saturday 12-1 (ST10)

Friday 4-6 (STL)

Wednesday 2-4 (S-version)

Matriculation number:_____

Name:_____

Signature:_____

INSTRUCTIONS TO CANDIDATES

1. This test contains a total of **TWELVE (12)** Questions.
2. Answer **ALL** questions.
3. The examination carries a total of 50 marks.
4. Candidates may use a help-sheet, up to half of A4 size (both sided).

Problem #	Your Grades
1 (4 points)	
2 (4 points)	
3 (4 points)	
4 (4 points)	
5 (4 points)	
6 (4 points)	
7 (4 points)	
8 (4 points)	
9 (5 points)	
10 (4 points)	
11 (4 points)	
12 (5 points)	
total (50 points)	

MA2108 Midterm**5 October****Name:** _____

Determine the limit of the following sequences. Justify your answer.

1. [4 marks] $\left\{ \frac{n^3 - 100n^2 + n - 58}{2n^3 + 65n + 69} \right\}$.

2. [4 marks] $\left\{ \left(\sqrt{4 - \frac{1}{n}} - 2 \right) n \right\}$.

Determine the limit of the following sequences. Justify your answer.

3. [4 marks] $\left\{ \frac{\sin(n^2)}{\sqrt{n}} \right\}$.

4. [4 marks] $\left\{ \frac{\sqrt[n]{(2n)!}}{n^2} \right\}$.

MA2108 Midterm**5 October****Name:** _____

Determine convergence or divergence of the following series. Justify your answer.

5. [4 marks] $\sum_{n=1}^{\infty} \frac{\sqrt{n+18}}{n^2+4n+3}$.

6. [4 marks] $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n+1}$.

Determine convergence or divergence of the following series. Justify your answer.

7. [4 marks] $\sum_{n=1}^{\infty} \frac{2^n(n!)}{n^n}$.

8. [4 marks] $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)[\ln(\ln n)]^{3/2}}$.

MA2108 Midterm**5 October****Name:** _____

Determine the absolute convergence, conditional convergence or divergence of the following series. Justify your answer.

9. [5 marks] $\sum_{n=1}^{\infty} \frac{(-1)^n(n+6)}{2n^2+1}$.

10. [4 marks] $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2n}{2n+1} \right)^{n(n+1)}$.

MA2108 Midterm**5 October****Name:** _____**11.** [4 marks] Find limit inferior and limit superior of the sequences:

$$\left\{ \sin\left(\frac{n\pi}{2}\right) + \cos(n\pi) \right\}.$$

Justify your answer.

12. [5 marks] Let $\{a_n\}$ be the sequence defined by

$$a_1 = 3 \quad a_{n+1} = 1 + \sqrt{a_n - 1}.$$

Does the sequence $\{a_n\}$ converges? If so, find its limit. Justify your answer.