

Tutorial Group:**Matriculation Number:****Name:****Signature:**

In order to get the full marks for the project of take-home exams, you need to have 50 or more points. Well there are 10 points in this take-home exam already.

Deadline: Monday, September 26. Hand in your papers to: the Mailbox labelled MA2108 on the 1st floor of S14. **Attach this front page to your solutions.**

Question 1. [2 points, 1 for each part]

- (a) If $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges, prove that $\sum_{n=1}^{\infty} (a_n + b_n)$ diverges.
 (b) If a and b are positive real numbers, prove that

$$\sum_{k=1}^{\infty} \frac{1}{(ak + b)^p}$$

converges if $p > 1$ and diverges if $p \leq 1$.

Question 2. [3 points, 1 for each part] Test the series for convergence or divergence.

- (a) $\sum_{k=1}^{\infty} (-1)^k 2^{1/k}$.
 (b) $\sum_{k=1}^{\infty} \frac{(-1)^k k}{(k+1)(k+2)}$.
 (c) $\sum_{k=1}^{\infty} (\sqrt[k]{2} - 1)$.

(Hint: Try the limit comparison test with the harmonic series. Use $\lim_{k \rightarrow \infty} \frac{2^{1/k} - 1}{1/k} = \lim_{x \rightarrow 0} \frac{2^x - 1}{x}$ and then use L'Hospital rule for finding the limit.)

Question 3 [5 points, 1 for each part]

Determine the absolute convergence, conditional convergence or divergence of each of the following series. Justify your answers.

- (a) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k} + 1}$.
 (b) $\sum_{k=1}^{\infty} \frac{(-1)^k}{2k^2 + (-1)^k}$.
 (c) $\sum_{k=1}^{\infty} \frac{\sin kt}{k^2 + 3}$, $t \in \mathbb{R}$.
 (d) $\sum_{k=2}^{\infty} \frac{(-1)^k \ln(\ln k)}{\sqrt{\ln k} + 1}$.
 (e) $\sum_{k=1}^{\infty} \frac{(-1)^k k^k}{(k+1)^k}$.