

Tutorial Group:**Matriculation Number:****Name:****Signature:**

In order to get the full marks for the project of take-home exams, you need to have 50 or more points. Well there are 10 points in this take-home exam already.

Deadline: Monday, November 7. Hand in your papers to: the Mailbox labelled MA2108 on the 1st floor of S14. **Attach this front page to your solutions.**

Question 1. [4 points, 1 for each part] For each of the following, determine all values of x for which the given series converges.

(a)
$$\sum_{k=1}^{\infty} \frac{(1-2x)^k}{k}.$$

(b)
$$\sum_{k=1}^{\infty} \frac{3^k}{k^3} (2x+1)^k.$$

(c)
$$\sum_{k=1}^{\infty} \frac{3^k x^k}{2^k (1-x)^k}, \quad x \neq 1.$$

(Hint: Let $t = \frac{3x}{2(1-x)}$.)

(d)
$$\sum_{k=1}^{\infty} \frac{1}{kx^k}, \quad x \neq 0.$$

(Hint: Let $t = \frac{1}{x}$.)

Question 2. [4 points, 1 for each part] Using any applicable method, find the Taylor series of each of the following functions at the indicated point, and specify the interval on which the series converges to the function.

(a) $f(x) = \cos x^2, \quad x_0 = 0.$

(b) $f(x) = \ln\left(\frac{1+x}{1-x}\right), \quad x_0 = 0.$

(c) $f(x) = \sqrt{x}, \quad x_0 = 1.$

(d) $f(x) = \frac{x^2}{1-x^2}, \quad x_0 = 0.$

Question 3. [2 points, 1 for each part]

(a) Use series to estimate the integral's value

$$\int_0^{0.1} \arctan x^2 dx$$

with an error of magnitude less than 10^{-8} .

(b) Let $f(x) = x^3 \sin x^9$. Find $f^{(30)}(0)$.