1. Find the limit inferior and limit superior of the following sequences
   a) $\left\{\dfrac{2-(-1)^n n}{4n+2}\right\}$.
   b) $\left\{\left(0.9+\sin\dfrac{n\pi}{2}\right)^n\right\}$.
   c) $\sqrt[n]{\dfrac{(n!)^2}{(2n)!}}$. (Hint: Use Proposition 8.15 of chapter 1 in the unpacked lecture notes.)
2. Let $\{a_n\}$ be a bounded sequence of real numbers. Show that
$$\limsup_{n\to\infty}\sqrt{|a_n|}=\sqrt{\limsup_{n\to\infty}|a_n|}.$$
   (Hint: Let $b_n=\sup\{|a_n|,|a_{n+1}|,|a_{n+2}|,\ldots\}$ and let
$$B_n=\sup\{\sqrt{|a_n|},\sqrt{|a_{n+1}|},\sqrt{|a_{n+2}|},\ldots\}.$$
   Recall from the definition that $\overline{\lim_{n\to\infty}}\sqrt{|a_n|}=\lim_{n\to\infty}B_n$ and $\overline{\lim_{n\to\infty}}|a_n|=\lim_{n\to\infty}b_n$. Prove that $B_n\le\sqrt{b_n}$ and $\sqrt{b_n}\le B_n$.)
3. Let $\{a_n\}$ and $\{b_n\}$ be Cauchy sequences. Show that $\{a_n+b_n\}$ and $\{a_nb_n\}$ are also Cauchy sequences.
4. For each of the following series, calculate the n-th partial sum S_n, and determine whether the series is convergent or divergent.
   i) $\displaystyle\sum_{n=1}^{\infty}\ln\frac{n+2}{n+3}$.
   ii) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$.