

1. Show that the series  $\sum_{n=1}^{\infty} (-1)^n \frac{\cos n}{2^n}$  is absolutely convergent.
2. For each of the following series, determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your answers.
  - (a).  $\sum_{n=1}^{\infty} (-1)^n \frac{3}{2n+1}$ .
  - (b).  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{4n+3}$ .
  - (c).  $\sum_{n=1}^{\infty} (-1)^n \left( \frac{1+2n}{3+4n} \right)^n$ .
  - (d).  $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\cos n}{n(\ln n)^2}$ .
3. Estimate the infinite sum  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^5}$  such that the error is within 0.001.
4. For each of the following sequence of functions, determine whether converges pointwise to a function, and find the limiting function if it exists. Justify your answers.
  - (a).  $\left\{ \left( 1 + \frac{x}{n} \right)^{nx} \right\}, x \in (-\infty, +\infty)$ .
  - (b).  $\{x^{n+1}\}, x \in [-1, 1]$ .
  - (c).  $\left\{ \frac{x^{2n}}{1+x^{2n}} \right\}, x \in [0, 1]$ .
5. Let  $\{F_n\}$  be a sequence of functions on an interval  $I$ . It is given that  $\{F_n\}$  converges uniformly on some function  $F$  on  $I$ . Suppose also that for each  $n \in \mathbb{Z}^+$ , there exists a real number  $M_n > 0$  such that

$$|F_n(x)| \leq M_n \quad \text{for all } x \in I.$$

Show that there exists a real number  $M$  such that  $|F(x)| \leq M$  for all  $x \in I$ .