1. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.

   (a). $F_n(x) = \dfrac{n^2}{n^2 + x^2}$, $x \in [0, 1]$.

   (b). $F_n(x) = x^n(1 - x)$, $x \in [0, 1]$.

   (c). $f_n(x) = \dfrac{n \ln x}{x^n}$, $x \in [1, \infty)$.

   (d). $f_n(x) = \dfrac{n \ln x \cos nx}{x^n}$, $x \in [4, \infty)$.

   (e). $F_n(x) = \dfrac{n^2}{n^2 + x^2}$, $x \in [0, +\infty)$.

2. Prove that each of the following series of functions converges uniformly on the indicated interval.

   i) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos nx}{n^2 + x^2}$, $x \in (-\infty, +\infty)$.

   ii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{1 + n^3x^2}$, $x \in [2, \infty)$.

   iii) $\displaystyle\sum_{n=1}^{\infty} \frac{xe^{-nx}}{n^2}$, $x \in (0, \infty)$.

3. Let $\displaystyle\sum_{n=1}^{\infty} f_n(x)$ and $\displaystyle\sum_{n=1}^{\infty} g_n(x)$ be series of functions on an interval I with

$$|f_n(x)| \leq g_n(x)$$

   for all $x \in I$ and $n \geq 1$. Suppose that the series of functions $\displaystyle\sum_{n=1}^{\infty} g_n(x)$ converges uniformly.

   Show that the series of functions $\displaystyle\sum_{n=1}^{\infty} f_n(x)$ converges uniformly.

4. Does the series of functions $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^x}$ converge uniformly on the interval $(0, +\infty)$?

   Justify your answer.