

MA2108

Midterm

11 October 2005

Time allowed: 1.5 hours

**Tutorial Group:**(circle one)

Monday 1-2 (ST1)

Monday 2-3 (ST2)

Monday 3-4 (ST3)

Monday 4-6 (ST4)

Wednesday 9-10 (ST5)

Wednesday 10-11 (ST6)

Wednesday 11-12 (ST7)

Wednesday 12-1 (ST8)

**Name:** \_\_\_\_\_

**Matriculation number:** \_\_\_\_\_

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**INSTRUCTIONS TO CANDIDATES**

1. This test contains a total of **SIX (6)** Questions.
2. Answer **ALL** six questions. Show your steps.
3. The examination carries a total of 50 marks.
4. Candidates may use a help-sheet, up to A4 size (both sided).

| Problem #            | Your Grades |
|----------------------|-------------|
| 1<br>(15 points)     |             |
| 2<br>(5 points)      |             |
| 3<br>(15 points)     |             |
| 4<br>(5 points)      |             |
| 5<br>(5 points)      |             |
| 6<br>(5 points)      |             |
| total<br>(50 points) |             |

**MA2108 Midterm****11 October****Name:** \_\_\_\_\_**1.** [15 marks] Determine the limit of the following sequences. Justify your answer.

(a)  $\left\{ \frac{n^3 - n - 28}{3n^3 + 65n^2 + 69} \right\}$ .

(b)  $\left\{ \frac{1}{\sqrt{9^n + 3^{n+2}} - \sqrt{9^n - n^2}} \right\}$ .

(c)  $\left\{ (n^2 + n)^{\frac{1}{1+2\ln n}} \right\}$ .

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2. [5 marks] Determine the limit of the sequence  $\left\{ \frac{\sqrt[n]{(3n)!}}{n^3} \right\}$ . Justify your answer.

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**3.** [15 marks] Determine convergence or divergence of the following series. Justify your answer.

(a) 
$$\sum_{n=1}^{\infty} \sqrt{\frac{3^n + 2^{n+1}}{5^n + n^2}}.$$

(b) 
$$\sum_{n=1}^{\infty} (-1)^n \frac{3n}{3n+1}.$$

(c) 
$$\sum_{n=1}^{\infty} \frac{[(2n)!]^{2^{n+2}}}{n!(3n)!}.$$

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4. [5 marks] Determine the absolute convergence, conditional convergence or divergence of the series

$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n + 2 \ln n}$ . Justify your answer.

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**5.** [5 marks] Let  $A$  be a non-empty bounded set of non-negative real numbers such that  $\sup A = 1$ . Consider the set

$$C = \{\sqrt{a + b + 2} : a, b \in A\}.$$

Find the value of  $\sup C$ . Justify your answer.

6. [5 marks] Let  $\{a_n\}$  be a sequence of positive real numbers such that  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{a_n}$  is convergent. Is it true that the series  $\sum_{n=1}^{\infty} \frac{a_n^3}{n^4}$  is divergent? Justify your answer.