

NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I

Take-home Exam 1

MA2108 Advanced Calculus II

Tutorial Group: _____

Name: _____ **Matric. No.:** _____

To be submitted during the lecture class on **Tuesday August 30, 2005**. Attach this sheet to your homework as cover page.

There will be a total of 4 homework during the semester.

The full score for each homework is 10 points.

Only your top 3 scores among the 4 homework will be used to count towards your final grade. Late homework will **NOT** be accepted.

1. Use the $\epsilon - N$ definition to prove that $\lim_{n \rightarrow \infty} \frac{3n^2 + 8}{2n^2 + 9} = \frac{3}{2}$.
2. For each of the following sequences, either find the limit or show that the limit does not exist.

(a) $\left\{ \left(\sqrt{n^2 + 2n} - n \right) \right\}$.

(b) $\left\{ (5^n + 3^n)^{\frac{1}{n}} \right\}$.

(c) $\left\{ \sqrt[5]{\frac{n! - 2n^5 + \ln n}{n! + 5^n + 3n}} \right\}$.

(d) $\left\{ \left(\frac{3n}{3n-1} \right)^{2n + \sqrt[3]{n}} \right\}$.

(e) $\left\{ \frac{n^{50} \cdot 50^n \cdot \cos n}{n!} \right\}$.

(f) $\left\{ \left(\sqrt{4 - \frac{3}{n}} - 2 \right) n \right\}$.

3. Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences. Prove that
 - (i) The sequence $\{|a_n - b_n|\}$ is convergent. [Hint: the function $f(x) = |x|$ is continuous.]
 - (ii) From (i) or otherwise, prove that the sequence $\{c_n\}$ defined by $c_n = \max\{a_n, b_n\}$ is also convergent. [Hint: First show that $\max\{a, b\} = \frac{1}{2}(a + b + |a - b|)$.]