

NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I

Take-home Exam 3

MA2108 Advanced Calculus II

Tutorial Group: _____

Name: _____ **Matric. No.:** _____

To be submitted during the lecture class on **Tuesday October 11, 2005**. Attach this sheet to your homework as cover page.

There will be a total of 4 homework during the semester.

The full score for each homework is 10 points.

Only your top 3 scores among the 4 homework will be used to count towards your final grade. Late homework will **NOT** be accepted.

Announcement. There will be a **test** on **Thursday October 11, 2005** during the lecture class. The test will cover materials from Chapter 1 to Chapter 2 of the lecture notes (or roughly, the topics covered in Tutorials 1-7).

The test is a **closed book test**, but you are allowed to bring along **ONE help sheet**.

Definition of a help sheet: A **help sheet** is a piece of paper of size not larger than A4 (21 cm by 30 cm). Anything on the help sheet must be **handwritten** and may be written on both sides of the paper. The handwriting can be as big or as small as the candidate may desire. However, the help sheet must **not** contain any machine printed information of any kind (such as photocopy of a page from either a book or handwritten notes.)

1. Determine the convergence or divergence of each of the following series. Justify your answers.

(i)
$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 2n - 1}.$$

(ii)
$$\sum_{n=1}^{\infty} \frac{1}{n(1 + 2 \ln n)}.$$

(iii)
$$\sum_{n=1}^{\infty} 2^n \left(1 - \frac{1}{n+1}\right)^{n^2}.$$

(iv)
$$\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}.$$

(v)
$$\sum_{n=1}^{\infty} (\sqrt[n]{3} - 1).$$
 [Hint: Try the limit comparison test with the harmonic series. Use

$$\lim_{n \rightarrow \infty} \frac{3^{1/n} - 1}{1/n} = \lim_{x \rightarrow 0} \frac{3^x - 1}{x}$$
 and then use L'Hospital rule for finding the limit.]

2. Determine the absolute convergence, conditional convergence or divergence of each of the following series. Justify your answers.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2\sqrt{n} + 1}.$$

(b)
$$\sum_{n=1}^{\infty} \frac{\cos(nt)}{n^2 + 1}, \quad t \in \mathbb{R}.$$

(e)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n^n}{(n+2)^n}.$$