NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2005/2006 Semester I Take-home Exam 3 MA2108 Advanced Calculus II

Tutorial Group:_________________________Name:_________________________Matric. No:_________________________

To be submitted during the lecture class on Tuesday October 11, 2005. Attach this sheet to your homework as cover page.

There will be a total of 4 homework during the semester.
The full score for each homework is 10 points.
Only your top 3 scores among the 4 homework will be used to count towards your final grade. Late homework will NOT be accepted.

Announcement. There will be a test on Thursday October 11, 2005 during the lecture class.
The test will cover materials from Chapter 1 to Chapter 2 of the lecture notes (or roughly, the topics covered in Tutorials 1-7).
The test is a closed book test, but you are allowed to bring along ONE help sheet.

Definition of a help sheet: A help sheet is a piece of paper of size not larger than A4 (21 cm by 30 cm). Anything on the help sheet must be handwritten and may be written on both sides of the paper. The handwriting can be as big or as small as the candidate may desire. However, the help sheet must not contain any machine printed information of any kind (such as photocopy of a page from either a book or handwritten notes.)

1. Determine the convergence or divergence of each of the following series. Justify your answers.
   (i) \( \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2 + 2n - 1} \).
   (ii) \( \sum_{n=1}^{\infty} \frac{1}{n(1 + 2 \ln n)} \).
   (iii) \( \sum_{n=1}^{\infty} 2^n \left( 1 - \frac{1}{n+1} \right)^n \).
   (iv) \( \sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n} \).
   (v) \( \sum_{n=1}^{\infty} (\sqrt{3} - 1) \). [Hint: Try the limit comparison test with the harmonic series. Use \( \lim_{n \to \infty} \frac{3^{1/n} - 1}{1/n} = \lim_{x \to 0} \frac{3^x - 1}{x} \) and then use L’Hospital rule for finding the limit.]

2. Determine the absolute convergence, conditional convergence or divergence of each of the following series. Justify your answers.
   (a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{2\sqrt{n} + 1} \).
   (b) \( \sum_{n=1}^{\infty} \frac{\cos(nt)}{n^2 + 1}, \quad t \in \mathbb{R} \).
   (c) \( \sum_{n=1}^{\infty} \frac{(-1)^n n^n}{(n + 2)^n} \).