1. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.
   (a) \( F_n(x) = x^2 + \frac{x}{n} \cos(nx), \quad x \in [-a, a], \quad a > 0. \)
   (b) \( F_n(x) = x^2 + \frac{x}{n} \cos(nx), \quad x \in \mathbb{R}. \) [Hint: Try to find a lower bound of \( T_n = \sup_{x \in \mathbb{R}} |F_n(x) - F(x)| \) by taking \( x = 2n\pi. \)]

2. Determine whether the following series of functions converge uniformly on the indicated intervals. Justify your answers.
   (a) \( \sum_{n=1}^{\infty} \frac{n \cos(nx)}{2n^3 + x^2}, \quad x \in (-\infty, \infty). \)
   (b) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n + x^2}, \quad x \in (-\infty, \infty). \) [Hint: Let \( S(x) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n + x^2} \) and let \( S_n(x) = \sum_{k=1}^{n} \frac{(-1)^k}{k + x^2}. \) Try to find an upper bound of \( T_n = \sup_{-\infty < x < \infty} |S_n(x) - S(x)| \) using alternating series estimation.]

3. Evaluate
   \[
   \sum_{n=0}^{\infty} \int_{0}^{\frac{1}{2}} \frac{x^n (1 - x^2)}{\sqrt{1 + x}} \, dx
   \]
   in simplest form. Justify your answer.

4. Find interval of convergence of each of the following power series:
   (a) \( \sum_{n=1}^{\infty} \frac{3^n}{n} \)(2x + 3)^n.
   (b) \( \sum_{n=0}^{\infty} \frac{1}{n^2} \left( \frac{x}{2} \right)^n. \) [Hint: Using root test to find the open interval in which the series converges absolutely. Then check the end-points.]