NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I Take-home Exam 4 MA2108 Advanced Calculus II

Tutorial Group:_____

Name:_____ **Matric. No:**_____

To be submitted during the lecture class on **Friday, October 28, 2005.** Attach this sheet to your homework as cover page.

There will be a total of 4 homework during the semester.
The full score for each homework is 10 points.
Only your top 3 scores among the 4 homework will be used to count towards your final grade. Late homework will **NOT** be accepted.

Announcement on Final Exam. The final exam. is a **closed book exam**, but you are allowed to bring along **TWO help sheets**.

Definition of a help sheet: A **help sheet** is a piece of paper of size not larger than A4 (21 cm by 30 cm). Anything on the help sheet must be **handwritten** and may be written on both sides of the paper. The handwriting can be as big or as small as the candidate may desire. However, the help sheet must **not** contain any machine printed information of any kind (such as photocopy of a page from either a book or handwritten notes.)

1. Determine whether the following sequences of functions converge uniformly on the indicated intervals. Justify your answers.

(a) $F_n(x) = x^2 + \dfrac{x}{n}\cos(nx), \quad x \in [-a, a], \quad a > 0.$

(b) $F_n(x) = x^2 + \dfrac{x}{n}\cos(nx), \quad x \in \mathbb{R}$. [Hint: Try to find a lower bound of $T_n = \sup_{x\in\mathbb{R}} |F_n(x) - F(x)|$ by taking $x = 2n\pi$.]

2. Determine whether the following series of functions converge uniformly on the indicated intervals. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n\cos(nx)}{2n^3 + x^2}, \quad x \in (-\infty, \infty).$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n + x^2}, \quad x \in (-\infty, \infty)$. [Hint: Let $S(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n + x^2}$ and let $S_n(x) = \displaystyle\sum_{k=1}^{n} \frac{(-1)^k}{k + x^2}$. Try to find an upper bound of $T_n = \sup_{-\infty<x<\infty} |S_n(x) - S(x)|$ using alternating series estimation.]

3. Evaluate

$$\sum_{n=0}^{\infty} \int_0^{\frac{1}{3}} \frac{x^n(1 - x^2)}{\sqrt{1 + x}}\, dx$$

in simplest form. Justify your answer.

4. Find interval of convergence of each of the following power series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n}(2x + 3)^n.$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n}\left(\frac{x}{2}\right)^{n^2}$. [Hint: Using root test to find the open interval in which the series converges absolutely. Then check the end-points.]