1. Find the interval of convergence of each of the following power series:
   
   i) \[ \sum_{n=1}^{\infty} \frac{(-2x)^n}{n^3}. \]
   
   ii) \[ \sum_{n=1}^{\infty} \frac{3^n (x-2)^n}{n+1}. \]
   
   iii) \[ \sum_{n=1}^{\infty} \frac{(1-3x)^n}{n}. \]

2. (a) By computing derivatives, find the Taylor series of \( f(x) = e^{2x} \) at \( x = 4 \). Write down also the Taylor polynomial \( T_3(f, 4) \).

   (b) Using the standard power series, find the Taylor series of the following functions at the indicated points:
   
   (i) \( f(x) = \sin^2(4x), \quad x_0 = 0 \),
   
   (ii) \( f(x) = \frac{1}{(x+1)(2x+1)}, \quad x_0 = 1 \). (Hint: Use partial fractions)

3. (i) By integrating from \( t = 0 \) to \( t = x \) the power series \( \frac{1}{1+t} = \sum_{n=0}^{\infty} (-1)^n t^n, \quad |t| < 1 \), show that
   
   \[ \ln(1 + x) = \sum_{n=1}^{\infty} \frac{(-1)^n+1 x^n}{n} \]
   
   for all \( |x| < 1 \).

   (ii) Use part (i) to find the Taylor series of \( \ln(1 + 2x^2) \) at \( x_0 = 0 \).

4. Use Taylor’s Theorem to show that
   
   \[ \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad \text{for all} \ x \in \mathbb{R}. \]

5. Use the method of power series and the alternating series estimation to estimate the integral’s value
   
   \[ \int_{0}^{0.2} \sin(x^2)dx. \]
   
   with an error of magnitude less than \( 10^{-8} \).