NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I — MA2108 Advanced Calculus II — Tutorial 10

1. Find the interval of convergence of each of the following power series:

   i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2x)^n}{n^{\frac{3}{2}}}$.

   ii) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{n+1}$.

   iii) $\displaystyle\sum_{n=1}^{\infty} \frac{(1-3x)^n}{n}$.

2. (a) By computing derivatives, find the Taylor series of $f(x) = e^{2x}$ at $x = 4$. Write down also the Taylor polynomial $T_3(f, 4)$.

   (b) Using the standard power series, find the Taylor series of the following functions at the indicated points:

   (i) $f(x) = \sin^2(4x), \quad x_0 = 0,$

   (ii) $f(x) = \dfrac{1}{(x+1)(2x+1)}, \quad x_0 = 1$. (Hint: Use partial fractions)

3. (i) By integrating from $t = 0$ to $t = x$ the power series $\dfrac{1}{1+t} = \displaystyle\sum_{n=0}^{\infty}(-1)^n t^n$, $|t| < 1$, show that

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}x^n}{n}$$

   for all $|x| < 1$.

   (ii) Use part (i) to find the Taylor series of $\ln(1+2x^2)$ at $x_0 = 0$.

4. Use Taylor's Theorem to show that

$$\cos x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} \quad \text{for all } x \in \mathbb{R}.$$

5. Use the method of power series and the alternating series estimation to estimate the integral's value

$$\int_0^{0.2} \sin(x^2)dx.$$

   with an error of magnitude less than 10^{-8}.