NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I — MA2108 Advanced Calculus II — Tutorial 5

1. For each of the following series, calculate the n-th partial sum S_n, and determine whether the series is convergent or divergent.

   (i) $\displaystyle\sum_{k=1}^{\infty} 3^k \pi^{-(k+2)}$.

   (ii) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n+2}{n+3}$.

   (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

2. Use the n-th term test for divergence (divergence test) to show that the following series are divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1-2^n}{2^{n+1}+n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} [(-1)^n + \cos \frac{n\pi}{2}]$.

3. Use the comparison test or limit comparison test to determine the convergence or divergence of each of the following series.

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+\sqrt{n}}{n^3-n+3}$.

   (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\sin n}{n^3}$.

   (iii) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+3}{3^{n+1}-n}$.

   (iv) $\displaystyle\sum_{n=1}^{\infty} \frac{4+(-1)^n}{6n}$.

   (v) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^{1+\frac{1}{n}}}$.

4. (a). Let $\displaystyle\sum_{n=1}^{\infty} a_n$ be a convergent series with each $a_n \geq 0$. Show that the series $\displaystyle\sum_{n=1}^{\infty} a_n^2$ is also convergent.
   [Hint: First show that $|a_n| < 1$ for n sufficiently large, and then use the comparison test. Another solution is to use limit comparison test.]

   (b). Give an example of a convergent series $\displaystyle\sum_{n=1}^{\infty} a_n$ such that $\displaystyle\sum_{n=1}^{\infty} \sqrt{a_n}$ diverges.