

NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I

MA2108 Advanced Calculus II

Tutorial 6

1. Use the integral test to determine the convergence or divergence of the series:

(a). 
$$\sum_{n=1}^{\infty} \frac{1}{n(1 + \ln n)}.$$

(b). 
$$\sum_{n=1}^{\infty} \frac{1}{n[1 + (\ln n)^2]}.$$

2. Use the ratio test to determine the convergence or divergence of each of the following series.

(a). 
$$\sum_{n=1}^{\infty} \frac{(3n)!}{6^n n! (2n)!}$$

(b). 
$$\sum_{n=1}^{\infty} a_n, \text{ where } a_1 = 1, a_n = 2 \left(1 - \frac{1}{n}\right)^n a_{n-1}, n = 2, 3, \dots$$

3. Use the (simplified) root test to determine the convergence or divergence of each of the following series.

(a). 
$$\sum_{n=1}^{\infty} \frac{5n^2 \cdot 3^n}{4^{n+4}}.$$

(b). 
$$\sum_{n=1}^{\infty} \frac{3^{2n}}{5^n} \left(1 - \frac{1}{2n}\right)^{n^2}.$$

(c). 
$$\frac{1}{4} + \frac{1}{5^2} + \frac{1}{4^3} + \frac{1}{5^4} + \frac{1}{4^5} + \frac{1}{5^6} + \frac{1}{4^7} + \frac{1}{5^8} + \dots$$

4. Determine the convergence or divergence of each of the following series. Justify your answers.

(i) 
$$\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{n^2}.$$

(ii) 
$$\sum_{n=1}^{\infty} \frac{n^4 + 1 + \ln n}{n + n^6 + 2}.$$

(iii) 
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.2}}.$$

(iv) 
$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n!} \cdot \frac{3^n}{5^n}.$$

(v) 
$$\sum_{n=1}^{\infty} \frac{1}{n(1 + \ln n)}.$$

(vi) 
$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln n}.$$