NATIONAL UNIVERSITY OF SINGAPORE

Department of Mathematics

2005/2006 Semester I MA2108 Advanced Calculus II Tutorial 7

1. (a) Use the alternating series test to show that the series
$$\sum_{n=2}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}(1+\ln n)^2}$$
is convergent.
(b) Denote the sum of the series in (a) by S. Estimate the error in the approximation
$$S \approx \sum_{n=2}^{100}(-1)^{n+1}\frac{1}{\sqrt{n}(1+\ln n)^2}.$$
2. Consider the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\ln n}{\sqrt{n}}$.
i) Use the alternating series test to show that the series is convergent.
ii) Using part i) or otherwise, show that the series is conditionally convergent.
3. Show that the series $\sum_{n=1}^{\infty}(-1)^{n}\frac{\cos n}{2^n}$ is absolutely convergent.
4. For each of the following series, determine whether the series is absolutely convergent, conditionally convergent or divergent. Justify your answers.
(a). $\sum_{n=1}^{\infty}(-1)^{n}\frac{3}{2n+1}$.
(b). $\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{4n+3}$.
(c). $\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{1+2n}{3+4n}\right)^{n}$.
(d). $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{\cos n}{n(\ln n)^2}$.
5. For each of the following sequence of functions, determine whether converges pointwise to a function, and find the limiting function if it exists. Justify your answers.
(a). $\left\{\left(1+\frac{x}{n}\right)^{nx}\right\}$, $x \in (-\infty, +\infty)$.
(b). $\{x^{n+1}\}$, $x \in [-1, 1]$.
(c). $\left\{\frac{x^{2n}}{1+x^{2n}}\right\}$, $x \in [0, 1]$.